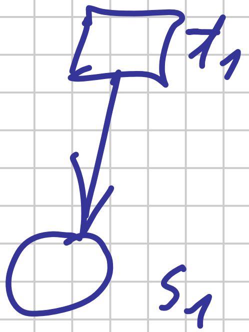
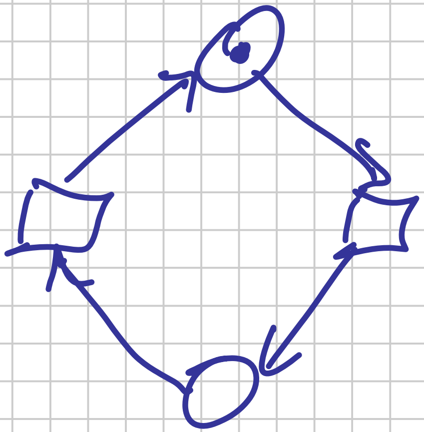
# Modellierung 27.10.10

Notiztitel 27.10.2010

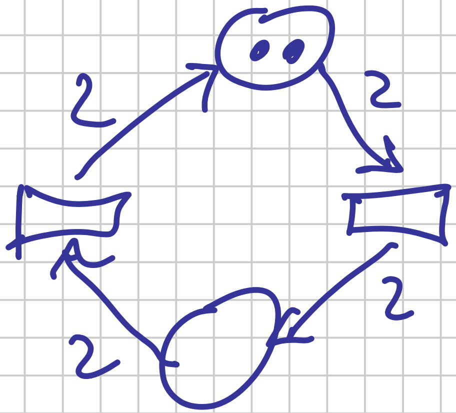



unbeschränkt

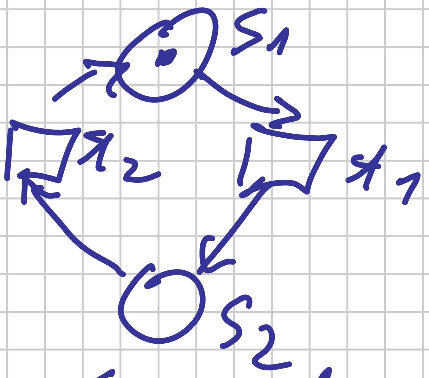
$(0) \xrightarrow{t_1} (1) \xrightarrow{t_1} (2) \xrightarrow{t_1} (3) \xrightarrow{t_1} (4) \rightarrow \ldots$

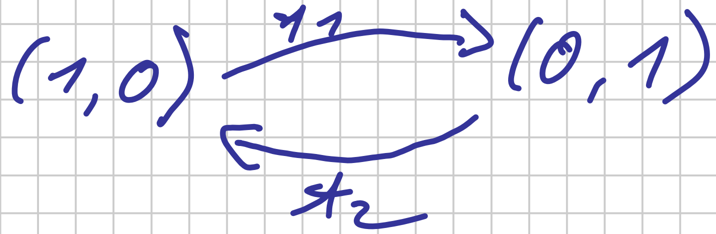


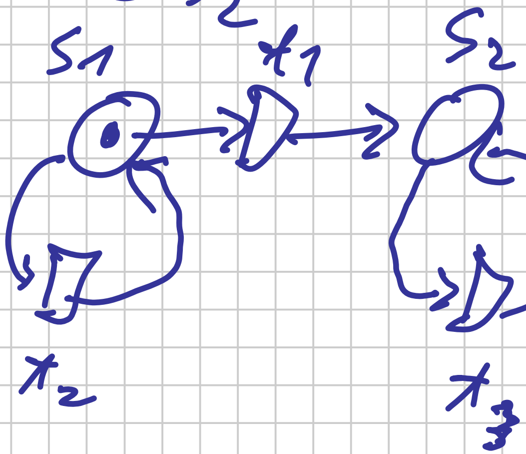
beschränkt, sicher

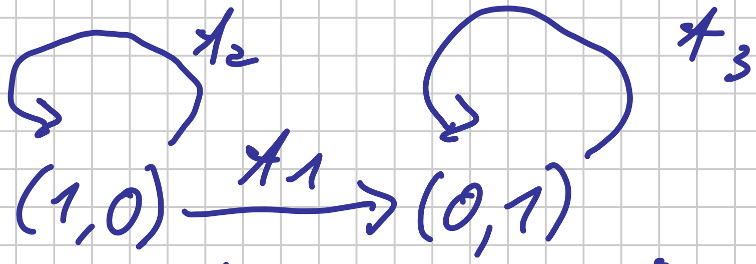



beschränkt, nicht sicher

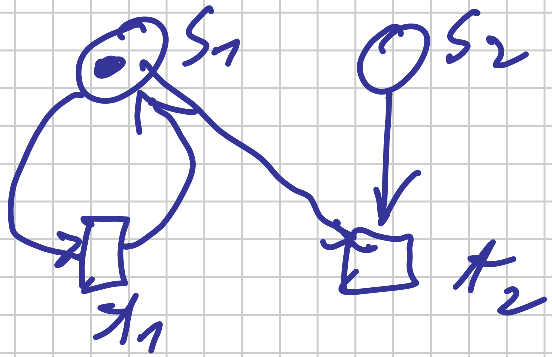


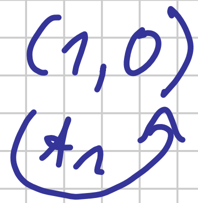



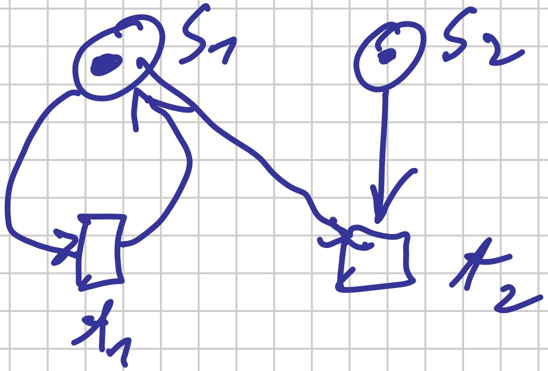




¬ stark lebendig
schwach lebendig
verklemmungsfrei




¬ schwach lebendig
verklemmungsfrei

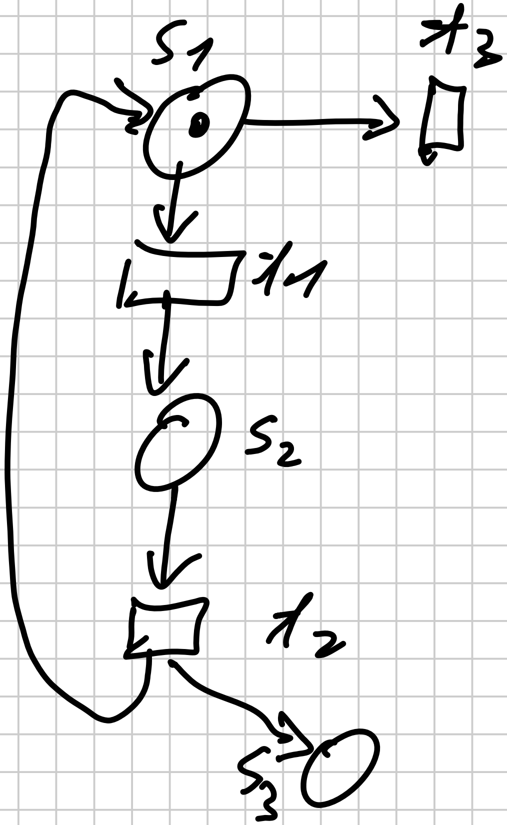


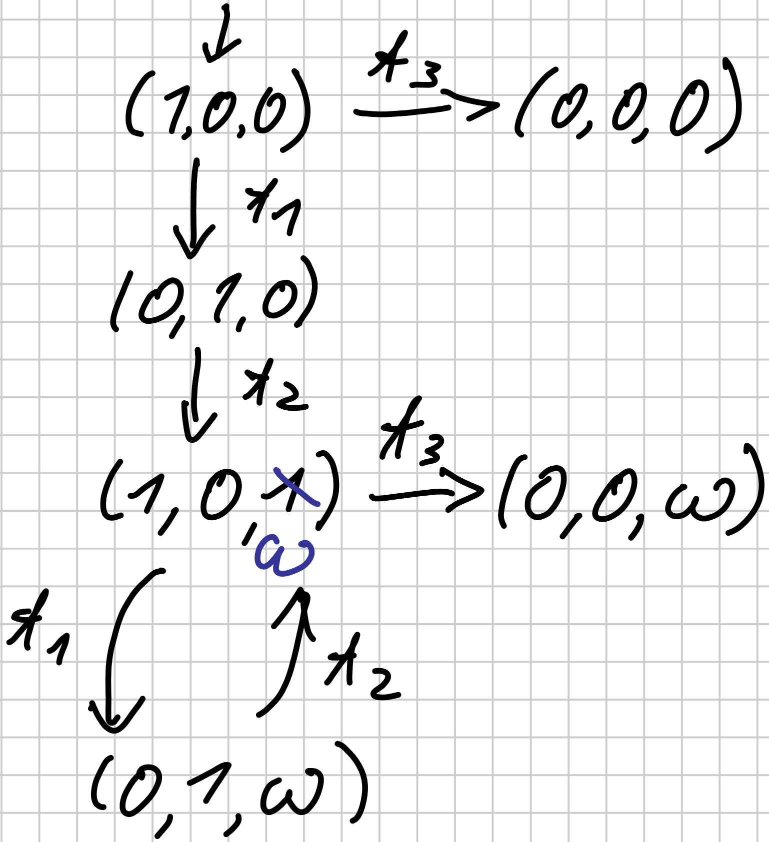

$$\begin{tikzcd}
(1,1) \arrow[loop below, "t_1"] \arrow[r, "t_2"] & (0,0)
\end{tikzcd}$$

$\neg$ stark lebendig

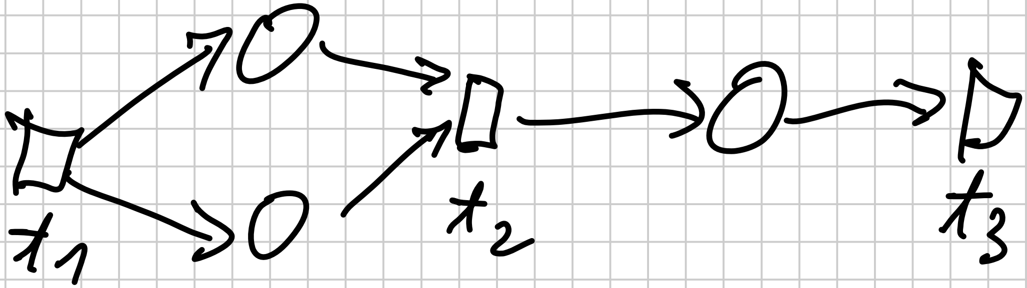
schwach lebendig

$\neg$ verklemmungsfrei


$s_1$
$t_3$
$t_1$
$s_2$
$t_2$
$s_3$

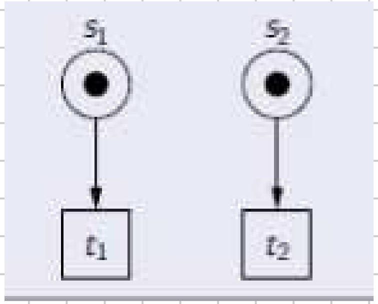

$(1,0,0)$
$t_3$
$(0,0,0)$
$t_1$
$(0,1,0)$
$t_2$
$(1,0,\cancel{1},\omega)$
$t_3$
$(0,0,\omega)$
$t_1$
$t_2$
$(0,1,\omega)$

Transitivität

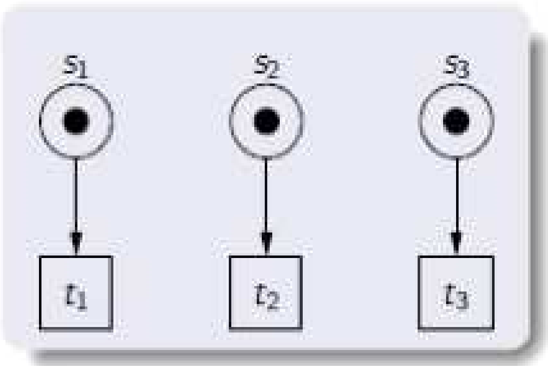


# Modellierung 3.11.10

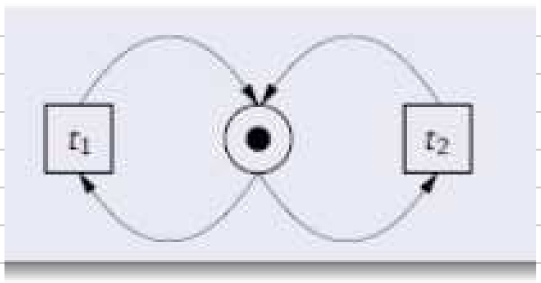
Notiztitel 03.11.2010




$$\begin{tikzcd}
(1,1) \arrow[r, "t_1"] \arrow[d, "t_2"] & (0,1) \arrow[d, "t_2"] \\
(1,0) \arrow[r, "t_1"] & (0,0)
\end{tikzcd}$$



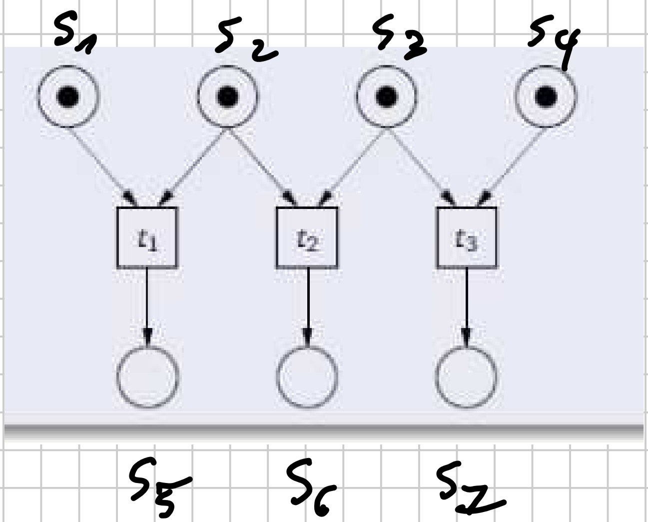

$$\begin{tikzcd}
(1,1,1) \arrow[rr, "t_1"] \arrow[dr, "t_3"] \arrow[dd, "t_2"] & & (0,1,1) \arrow[dr, "t_3"] \arrow[dd, "t_2"] & \\
& (1,1,0) \arrow[rr, "t_1"] \arrow[dd, "t_2"] & & (0,1,0) \arrow[dd, "t_2"] \\
(1,0,1) \arrow[rr, "t_1"] \arrow[dr, "t_3"] & & (0,0,1) \arrow[dr, "t_3"] & \\
& (1,0,0) \arrow[rr, "t_1"] & & (0,0,0)
\end{tikzcd}$$




$$\bullet t_1 = (1)$$

$$\bullet t_2 = (1)$$

$$\bullet t_1 \oplus \bullet t_2 = (2) \not\leq (1)$$

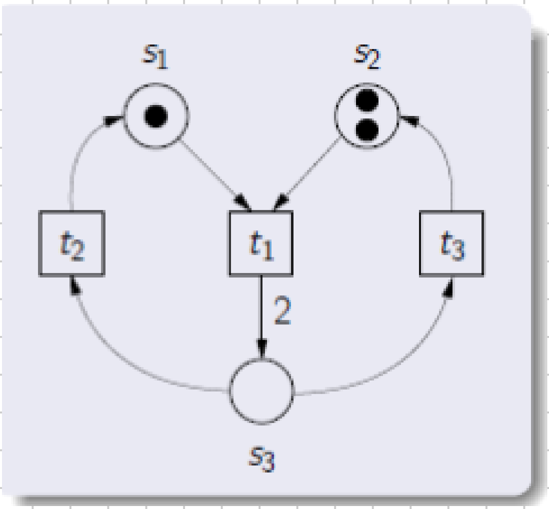



$$\vec{m} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad \vec{u} + \vec{v} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}$$

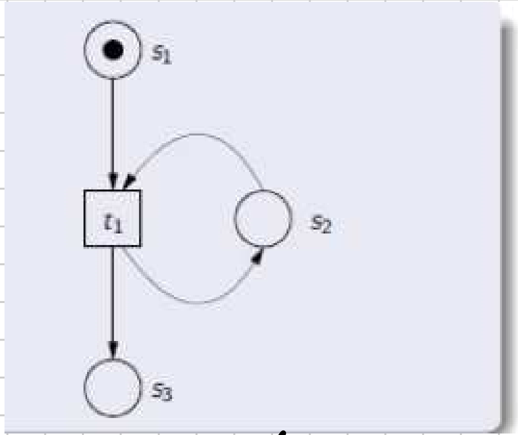
$$C \cdot \vec{u} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} c_{11} \cdot u_1 + c_{12} \cdot u_2 + c_{13} \cdot u_3 \\ c_{21} \cdot u_1 + c_{22} \cdot u_2 + c_{23} \cdot u_3 \end{pmatrix}$$

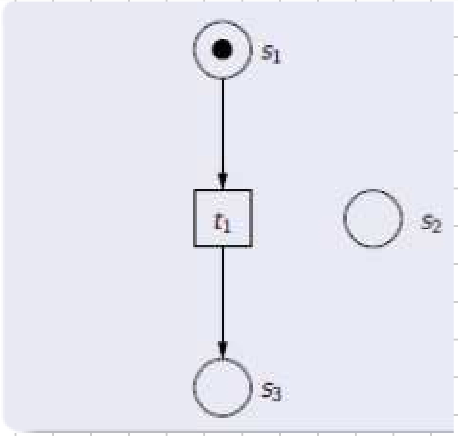
$$u \cdot C = (u_1, u_2) \cdot \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{pmatrix} = (u_1 c_{11} + u_2 c_{21}, u_1 c_{12} + u_2 c_{22}, u_1 c_{13} + u_1 c_{23})$$




| | $t_1$ | $t_2$ | $t_3$ |
|---|---|---|---|
| $s_1$ | 0 − 1 | 1 − 0 | 0 − 0 |
| $s_2$ | 0 − 1 | 0 − 0 | 1 − 0 |
| $s_3$ | 2 − 0 | 0 − 1 | 0 − 1 |

$$C = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix}$$




$$\begin{array}{cc} & t_1 \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} & \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

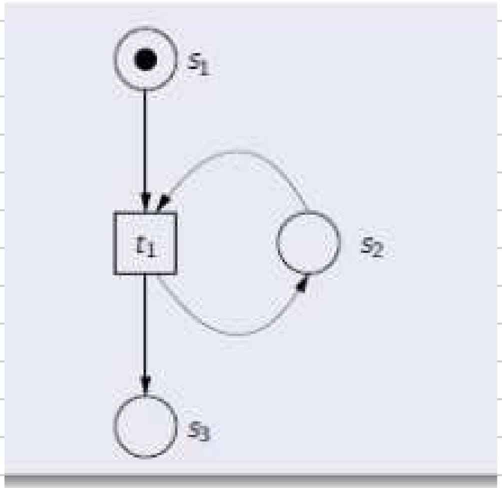
$$\begin{array}{cc} & t_1 \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} & \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

$$C = \begin{array}{c|c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} & \\ \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} & \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} \end{array} \qquad \vec{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \qquad \vec{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$C \cdot \vec{u}_1 = \begin{pmatrix} -1 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 \\ -1 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 \\ 2 \cdot 1 + (-1) \cdot 0 + (-1) \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \qquad C \cdot \vec{u}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

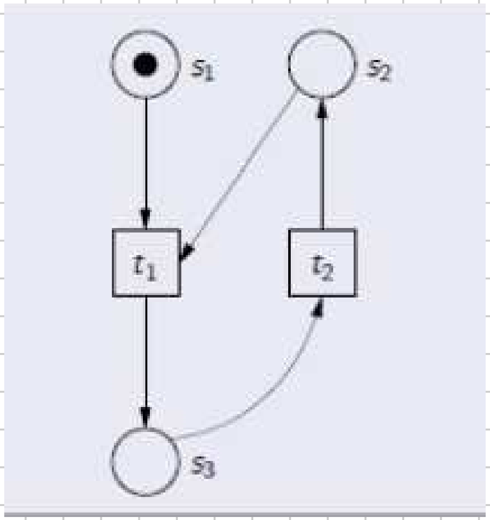
$$\vec{u} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$$

$$\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 2 + 1 \cdot 2 + 0 \cdot 1 \\ -1 \cdot 2 + 0 \cdot 2 + 1 \cdot 1 \\ 2 \cdot 2 + -1 \cdot 2 + -1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$



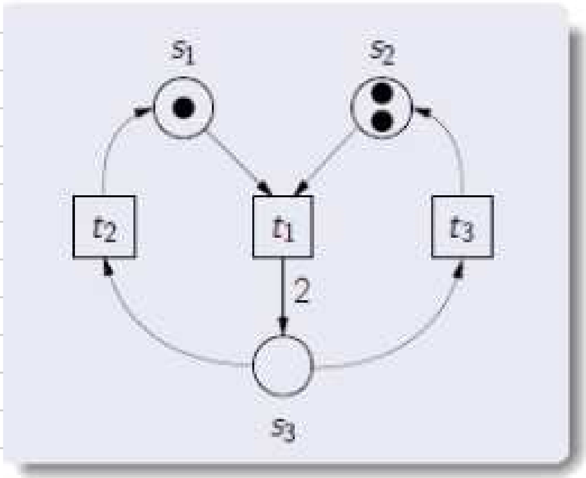

$$C = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array}\begin{array}{c} t_1 \\ \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} (1) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$




$$C = \begin{pmatrix} -1 & 0 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + -1 \cdot 1 + 0 \\ 0 + -1 \cdot 1 + 1 \cdot 1 \\ 0 + 1 \cdot 1 + -1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$



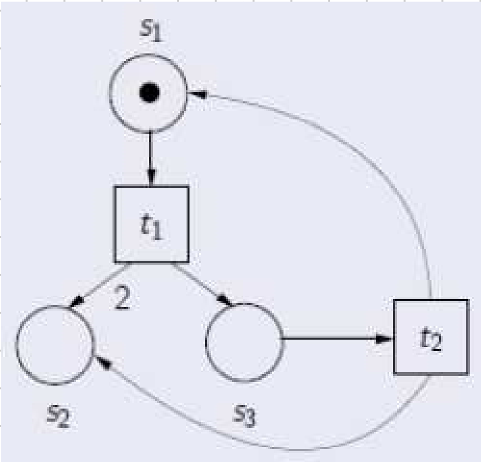

$$\begin{pmatrix}2\\2\\0\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \begin{pmatrix}-1 & 1 & 0\\ -1 & 0 & 1\\ 2 & -1 & -1\end{pmatrix} \cdot \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$$

$$\begin{aligned}
2 &= 1 - u_1 + u_2 \\
2 &= 2 - u_1 + u_3 \\
0 &= 0 + 2u_1 - u_2 - u_3 \\
\hline
4 &= 3 \quad \lightning
\end{aligned}$$

Markierung $\begin{pmatrix}2\\2\\0\end{pmatrix}$ nicht erreichbar!

# Modellierung 10.11.10

Notiztitel 10.11.2010




$$\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{array}{cc} & \begin{matrix} t_1 & \quad t_2 \end{matrix} \\ \begin{matrix} s_1\\ s_2\\ s_3\end{matrix} & \begin{pmatrix}-1&1\\2&1\\1&-1\end{pmatrix}\end{array} \cdot \begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}1\\20\\0\end{pmatrix}$$

$$1 - u_1 + u_2 = 1$$

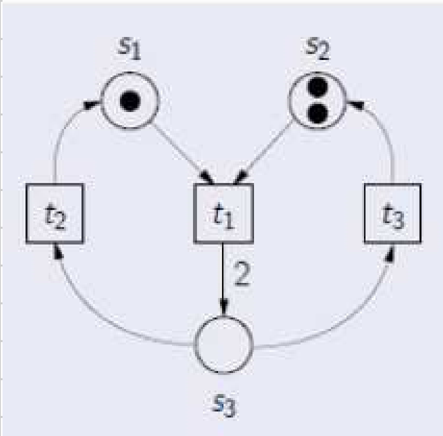
$$2u_1 + u_2 = 20 \quad | \quad 3u_1 = 20$$

$$u_1 - u_2 = 0 \quad | \quad u_1 = u_2$$

$$(1,0,0) \xrightarrow{t_1} (0,2,1) \xrightarrow{t_2} (1,\cancel{3}\,\omega,0)$$

$$(1,\omega,0) \xrightarrow{t_1} (0,\omega,1) \xrightarrow{t_2} (1,\omega,0)$$

# T-Invariante



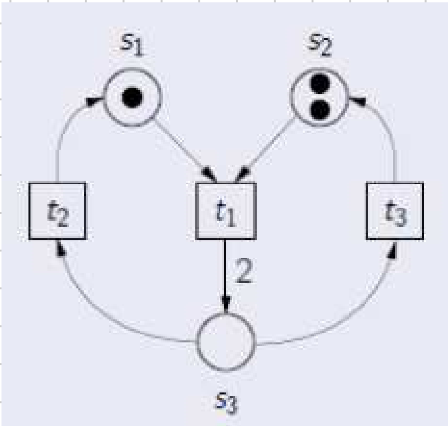

$$
\begin{array}{c c}
 & \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\
\begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} & \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix}
\end{array}
\cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
$$

$$
\begin{aligned}
-u_1 + u_2 \qquad &= 0 \quad | \; u_1 = u_2 \\
-u_1 \qquad + u_3 &= 0 \quad | \; u_1 = u_3 = u_2 \\
2u_1 - u_2 - u_3 &= 0
\end{aligned}
$$

$$
\vec{u} = k \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} k \\ k \\ k \end{pmatrix}
$$

$$
\begin{tikzcd}
(1,2,0) \arrow[d, "t_1"] & \\
(0,1,2) \arrow[r, "t_3"] \arrow[d, "t_2"] & (0,2,1) \arrow[ul, bend right, "t_2"'] \\
(1,1,1) \arrow[uu, bend left, "t_3"] &
\end{tikzcd}
$$

S-Invariante




$$(v_1\ v_2\ v_3) \cdot \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} = (0\ 0\ 0)$$

$$-v_1 - v_2 + 2v_3 = 0$$

$$v_1 - v_3 = 0 \quad | \quad v_1 = v_3$$

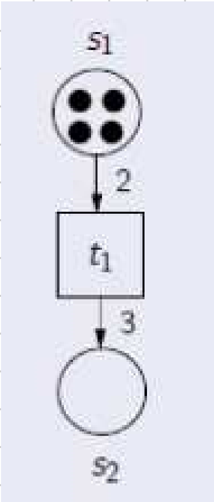
$$v_2 - v_3 = 0 \quad | \quad v_2 = v_3$$

$$v = (v_1\ v_1\ v_1) = v_1 \cdot (1\ 1\ 1)$$

$$v = (1\ 1\ 1)$$

$$\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

$$v \cdot \vec{m} = m_1 + m_2 + m_3 = 3 = v \cdot m_0$$




$$(v_1,\ v_2)\cdot\begin{pmatrix}-2\\3\end{pmatrix} = (0)$$

$$-2v_1 + 3v_2 = 0$$

$$v_1 = \frac{3}{2}v_2$$

$$v = v_2\cdot\left(\frac{3}{2}\ \ 1\right)$$

$$v = \ell\cdot(3\ \ 2)$$

$$v\cdot\vec{m} = 3\cdot m_1 + 2\cdot m_2 = 12 = v\cdot\vec{m_0}$$

$$m = (0,5)$$

$$v\cdot m = 3\cdot 0 + 2\cdot 5 = 10 \neq 12$$

# Modellierung 17.11.10

Notiztitel 17.11.2010

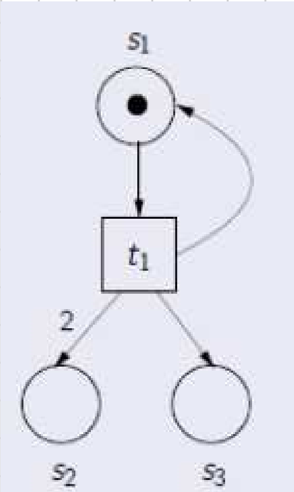
$$v \cdot \begin{array}{c|c} & \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} & \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$$v_1 - v_3 = 0 \quad | \; v_1 = v_3$$
$$-v_1 - v_2 + v_3 + v_4 = 0$$
$$v_2 - v_4 = 0 \quad | \; v_4 = v_2$$
$$-v_4 + v_5 = 0 \quad | \; v_4 = v_5$$
$$v_1 = v_3, \quad v_2 = v_4 = v_5$$

$$v = (a\, b\, a\, b\, b) = a(1\,0\,1\,0\,0) + b(0\,1\,0\,1\,1)$$

$$v \cdot \vec{m} = (a\, b\, a\, b\, b) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = a + 2b \quad \Big| \quad \vec{v} \cdot \vec{m}_0 = (a\, b\, a\, b\, b) \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} =$$

$$\neq \quad \rightarrow a + b$$




$$(v_1, v_2, v_3) \cdot \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array} \begin{array}{c} t_1 \\ \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \end{array} = 0 \Rightarrow 2v_2 + v_3 = 0$$

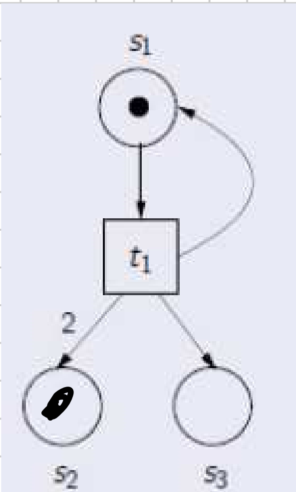
$$v_3 = -2v_2$$

$$v = \ell(0 \;\; 1 \; -2)$$

$$v \cdot \vec{m} = 0m_1 + m_2 - 2m_3 = 0 \Rightarrow m_2 = 2m_3$$

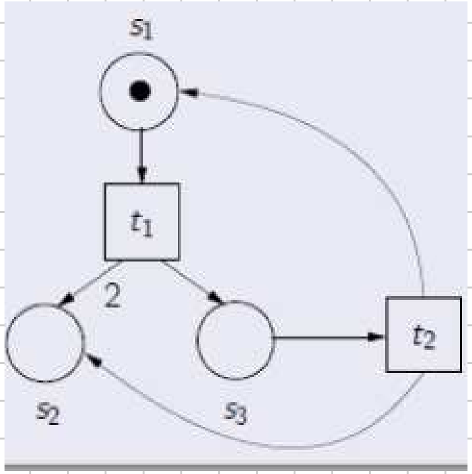
$$v \cdot \vec{m}_0 = 0$$

---




$$v \cdot \vec{m}_0 = 0m_1 + m_2 - 2m_3 = 1 = v \cdot \vec{m}$$

für alle erreichb. Mark. $m_2 = 2m_3 + 1$




$$\begin{array}{c|cc} & t_1 & t_2 \\ \hline s_1 & & \\ s_2 & & \\ s_3 & & \end{array}\qquad \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \end{array}\begin{array}{c} t_1 \quad t_2 \\ \begin{pmatrix} -1 & 1 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}\end{array}$$

$$-v_1 + 2v_2 + v_3 = 0$$
$$v_1 + v_2 - v_3 = 0$$

---

$$v_2 = 0$$
$$v_1 = v_3$$

$$v = l \cdot (1\ 0\ 1)$$

$$v \cdot \vec{m}_0 = (1\ 0\ 1)\begin{pmatrix}1\\0\\0\end{pmatrix} = 1$$

$$\|$$

$$v \cdot \vec{m} = (1\ 0\ 1)\begin{pmatrix}1\\2\ 0\\0\end{pmatrix} = 1$$

gemäß S-Invariante erreichbar, tatsächlich aber nicht

$$
\begin{array}{c|cccc}
 & t_1 & nt_1 & t_2 & nt_2 \\
\hline
K_1 & & & & \\
NK_1 & & & & \\
K_2 & & & & \\
NK_2 & & & & \\
S & & & &
\end{array}
\qquad
\begin{pmatrix}
1 & -1 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & -1 & 1 \\
-1 & 1 & -1 & 1
\end{pmatrix}
$$

S-Invarianten

$$
\begin{aligned}
v_1 - v_2 \qquad\qquad\qquad - v_5 &= 0 \\
-v_1 + v_2 \qquad\qquad\qquad + v_5 &= 0 \\
v_3 - v_4 - v_5 &= 0 \\
-v_3 + v_4 + v_5 &= 0
\end{aligned}
$$

$$
\left.\begin{aligned}
v_1 &= v_2 + v_5 \\
v_3 &= v_4 + v_5
\end{aligned}\right\}
\quad (v_2+v_5,\ v_2,\ v_4+v_5,\ v_4,\ v_5)
$$

$$= v_2\cdot(1,1,0,0,0) + v_4\cdot(0,0,1,1,0) + v_5\cdot(1,0,1,0,1)$$

wähle $v_2 = v_4 = 0$ $\quad (1\,0\,1\,0\,1)\cdot\vec{m} = m_1 + m_3 + m_5 = 1 = v\cdot\vec{m}_0$

Annahme: $m_1 \geq 1,\ m_3 \geq 1 \Rightarrow v\cdot\vec{m} = m_1 + m_3 + m_5 \geq 2$

$\lightning$ Widerspruch

# Speisende Philosophen

eine der S-Invarianten

$$v = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$
$$\quad F_1 \; F_2 \; H_1 \; H_2 \; W_1 \; W_2 \; E_1 \; E_2$$

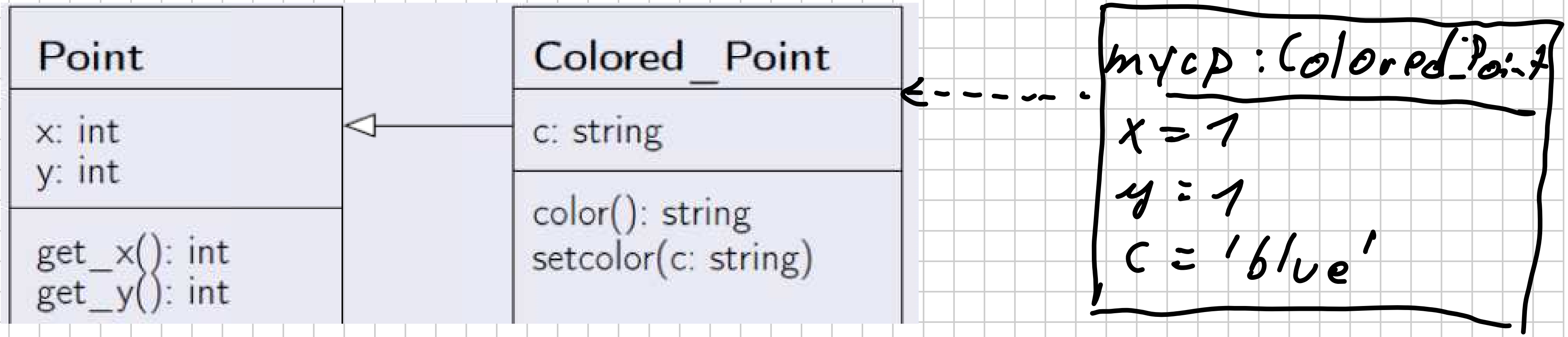
$$v \cdot \vec{m}_0 = 1$$

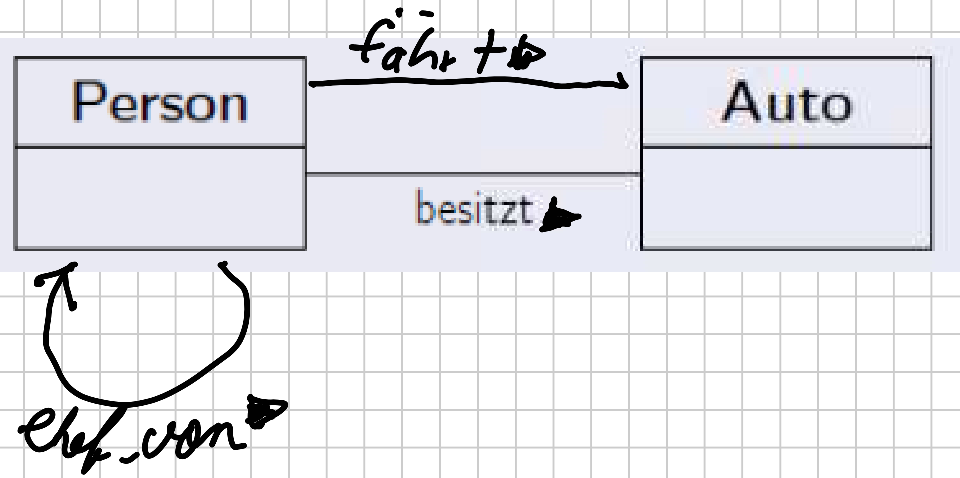
Verklemmung: beide in $W_1$ bzw $W_2$ $\quad v \cdot \vec{m} = 2 \neq 1$

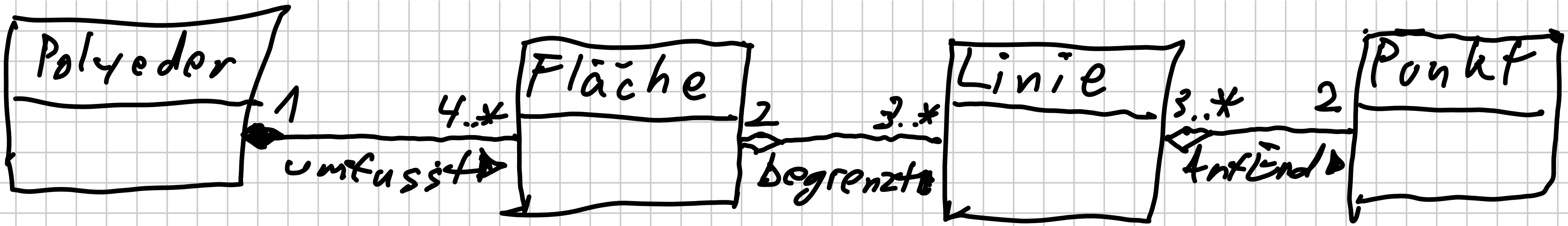
$\Rightarrow$ Verklemmung kann nicht auftreten

# Modellierung 1.12.10

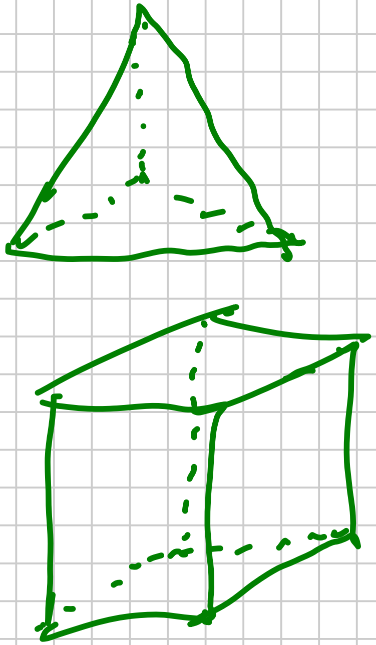
Notiztitel 01.12.2010

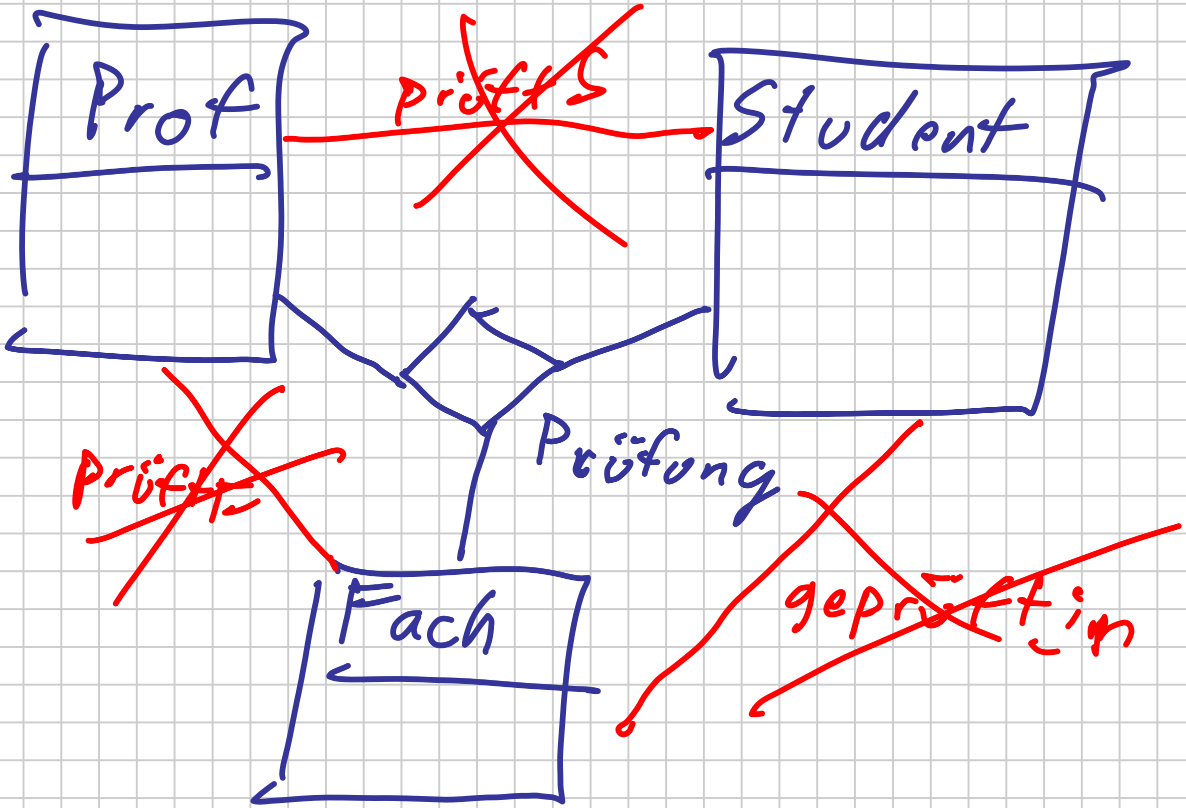


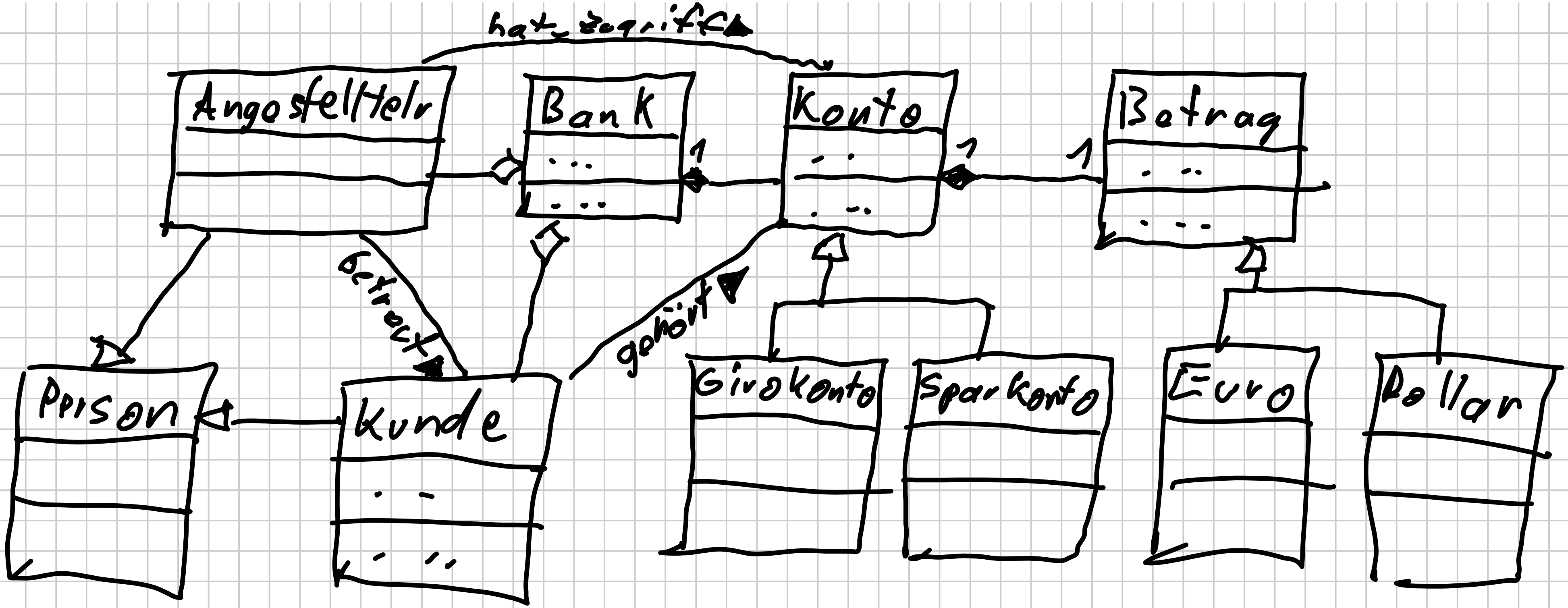


Polyeder
1
umfasst
4..*
Fläche
2
begrenzt
3..*
Linie
3..*
2
Punkt
Anf/End



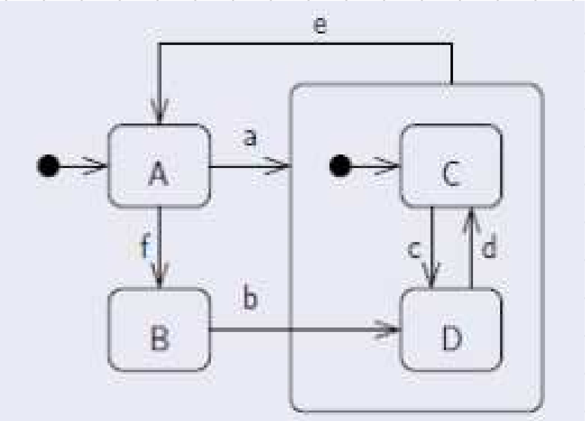
Prof
~~prüft~~
Student
Prüfung
~~prüft~~
~~geprüft in~~
Fach

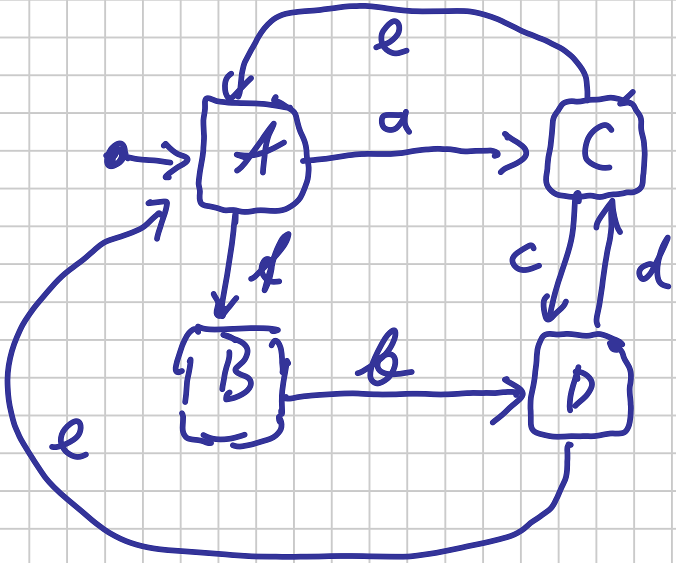


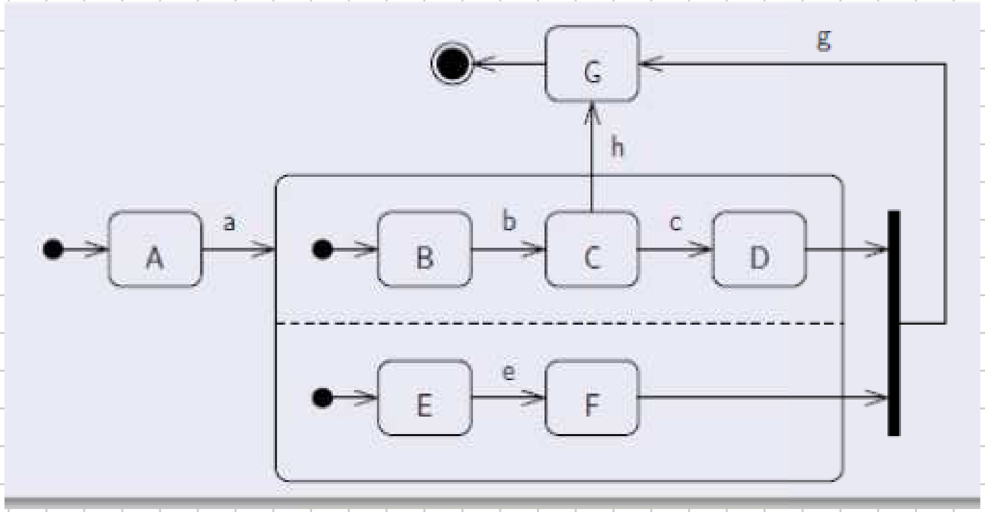
hat_zugriff
Angestellter
Bank
...
...
1
Konto
..
...
1
1
Betrag
...
...
betreut
gehört
Person
Kunde
..
...
Girokonto
Sparkonto
Euro
Dollar

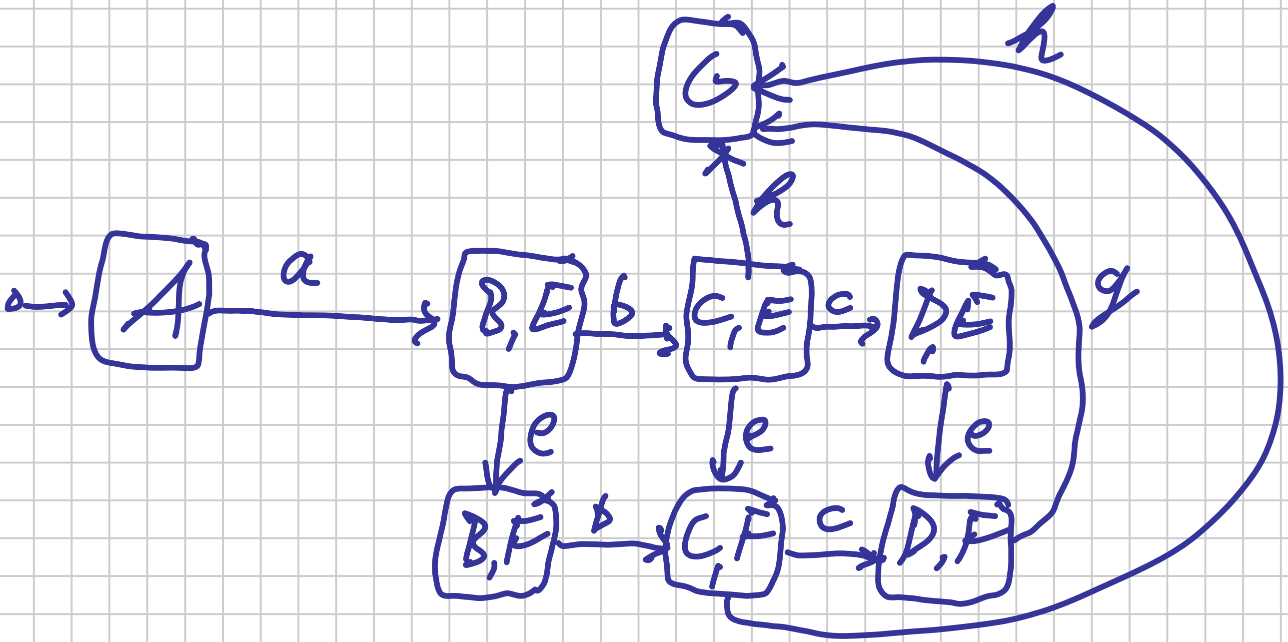
# Modellierung 22.12.10

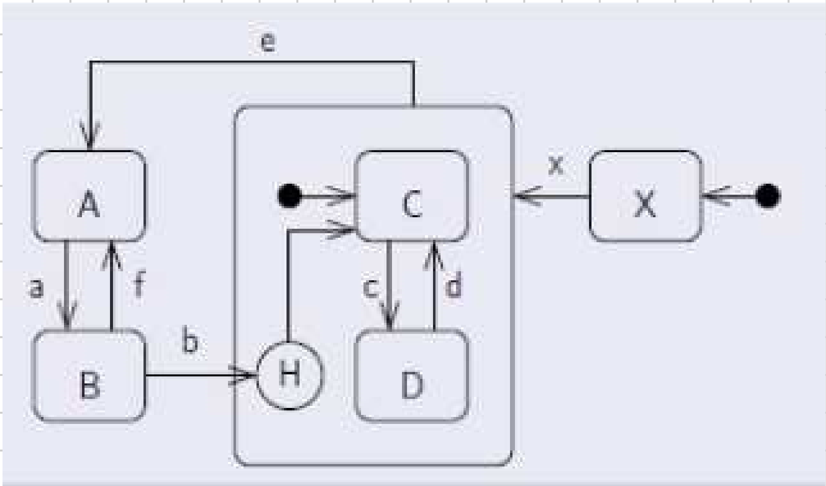
Notiztitel 22.12.2010

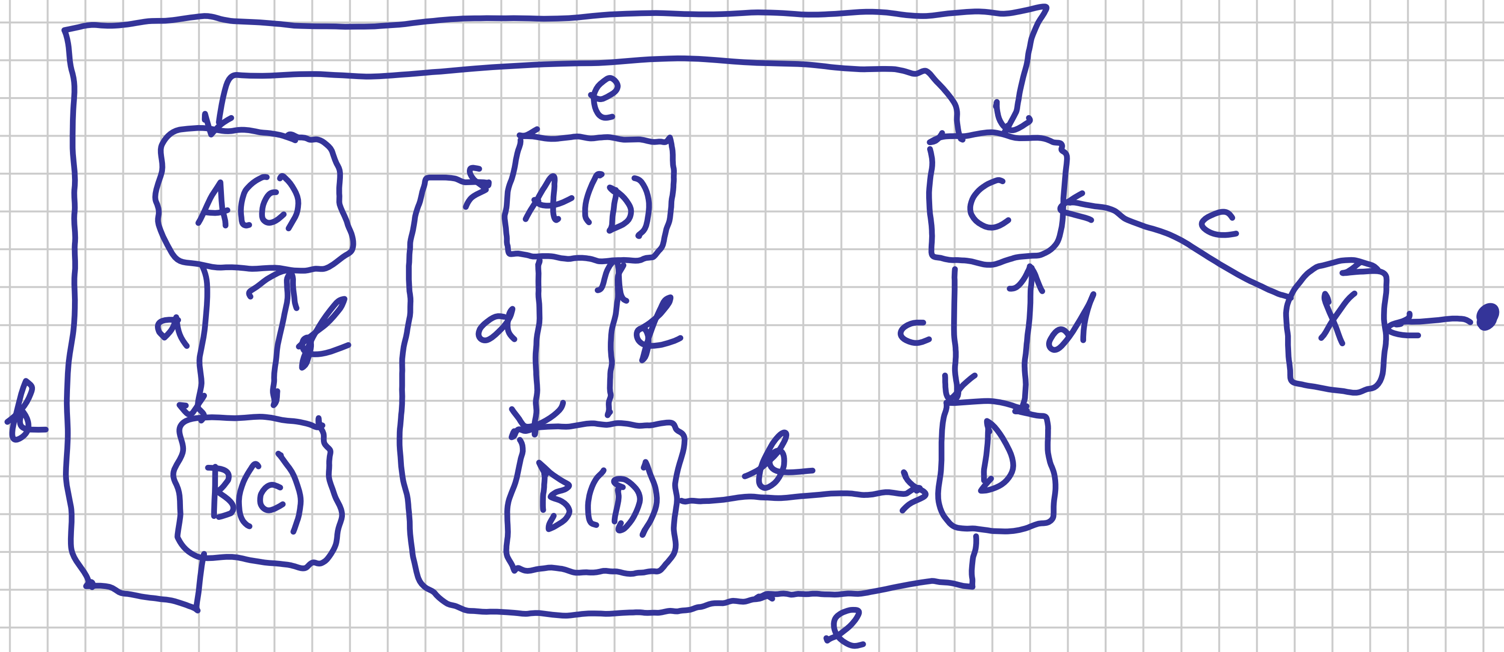



G
g
h
A
a
B
b
C
c
D
e
E
F

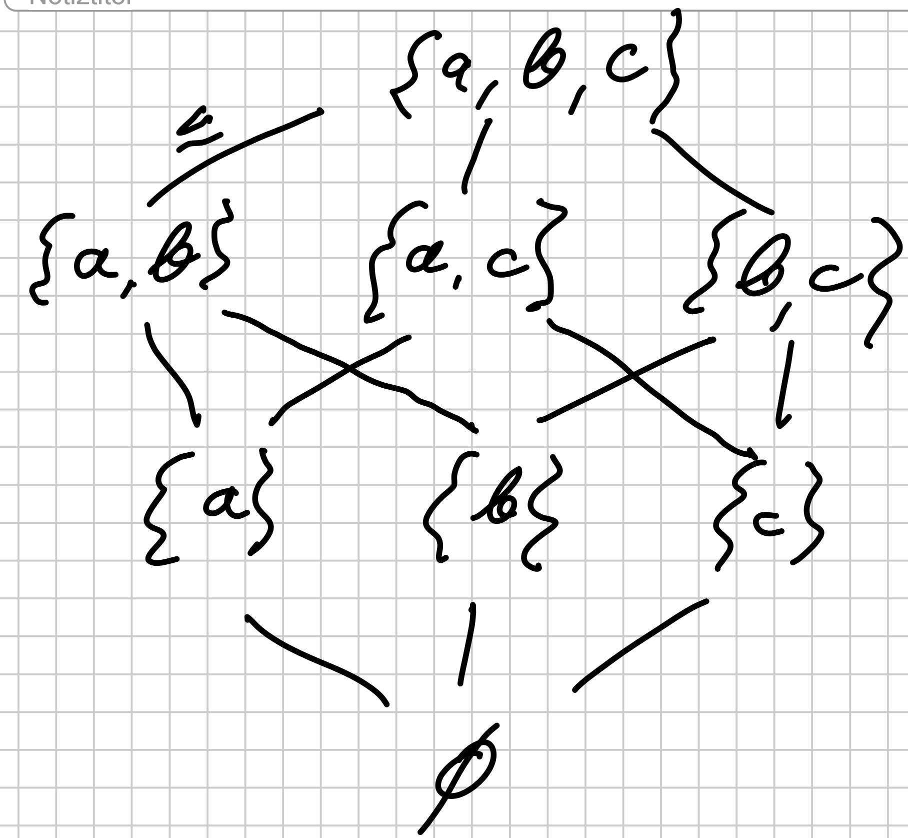

G
h
h
g
A
a
B,E
b
C,E
c
D,E
e
e
e
B,F
b
C,F
c
D,F


e
A
C
x
X
a
f
c
d
b
B
H
D


e
A(C)
A(D)
C
c
X
a
f
a
f
c
d
b
B(C)
B(D)
b
D
e

# Modellierung 12.1.11

Notiztitel 12.01.2011




Partielle Ordnung:

$\subseteq$ auf Mengen

Reflexivität: $M \subseteq M$

Transitivität:

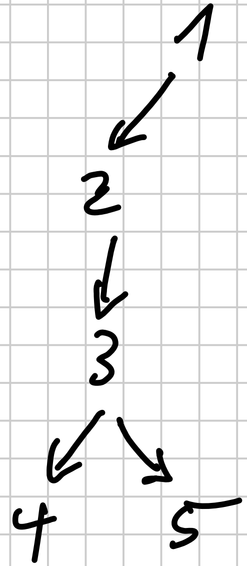
$A \subseteq B,\ B \subseteq C \Rightarrow A \subseteq C$

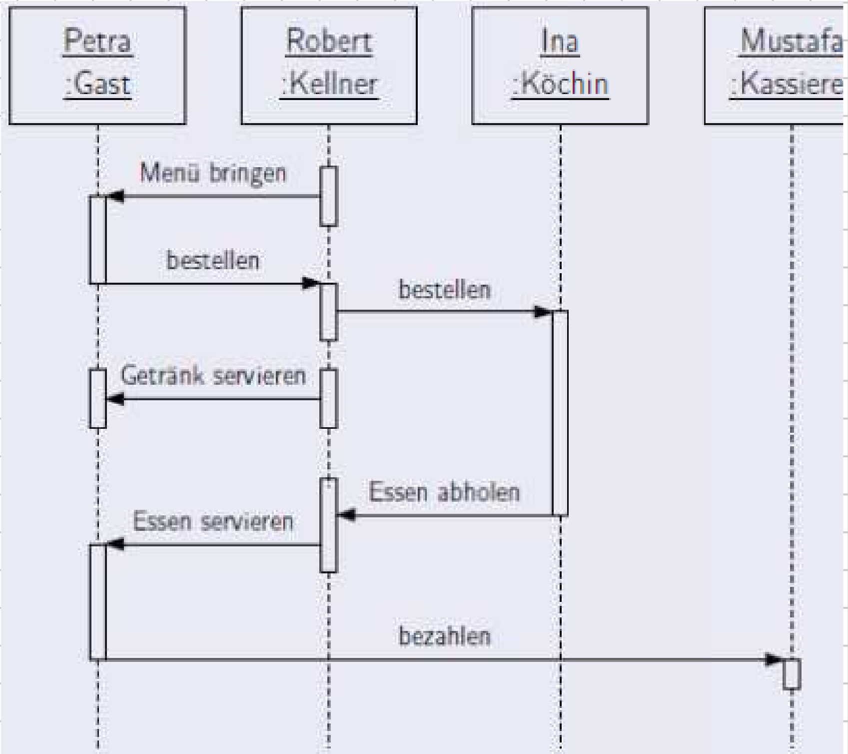
Antisymmetrie:

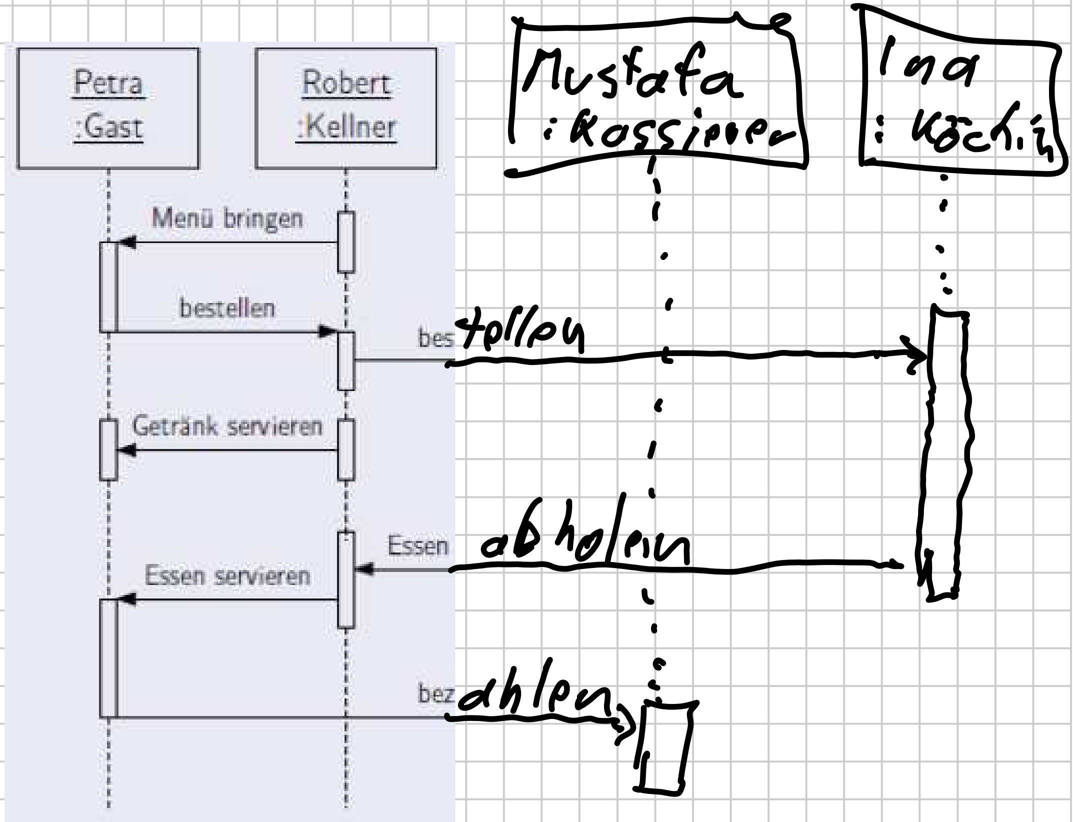
$A \subseteq B,\ B \subseteq A \Rightarrow A = B$

$R = \{(1,2),(2,3),(3,4),(3,5)\}$

$R^* = \{(1,2),(2,3),(3,4),(3,5),$
$(1,3),(1,4),(1,5),$
$(2,4),(2,5),(1,1),(2,2)$
$(3,3),(4,4),(5,5)\}$




Petra :Gast
Robert :Kellner
Ina :Köchin
Mustafa :Kassierer
Menü bringen
bestellen
bestellen
Getränk servieren
Essen abholen
Essen servieren
bezahlen


Petra :Gast
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Menü bringen
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Essen servieren
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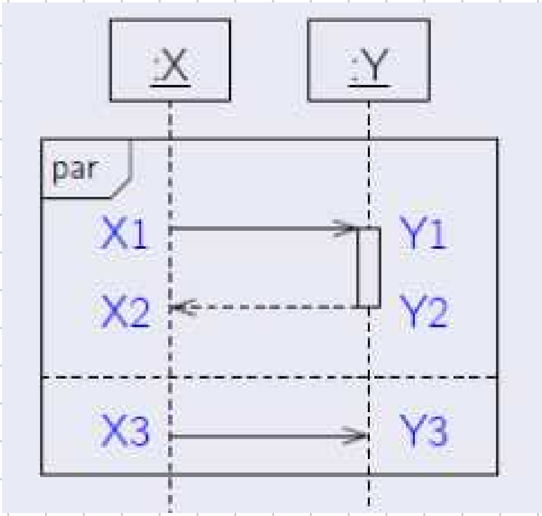



Ordnung auf den Ereignissen

$X1 < Y1 < Y2 < X2$

$X3 < Y3$

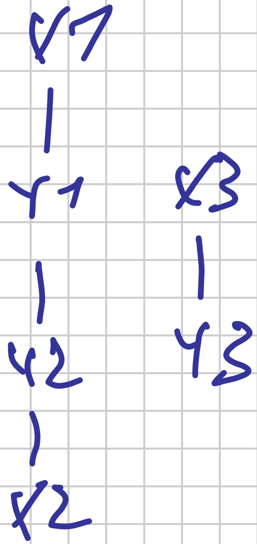
mögliche Abläufe:

$X1, Y1, Y2, X2, X3, Y3$

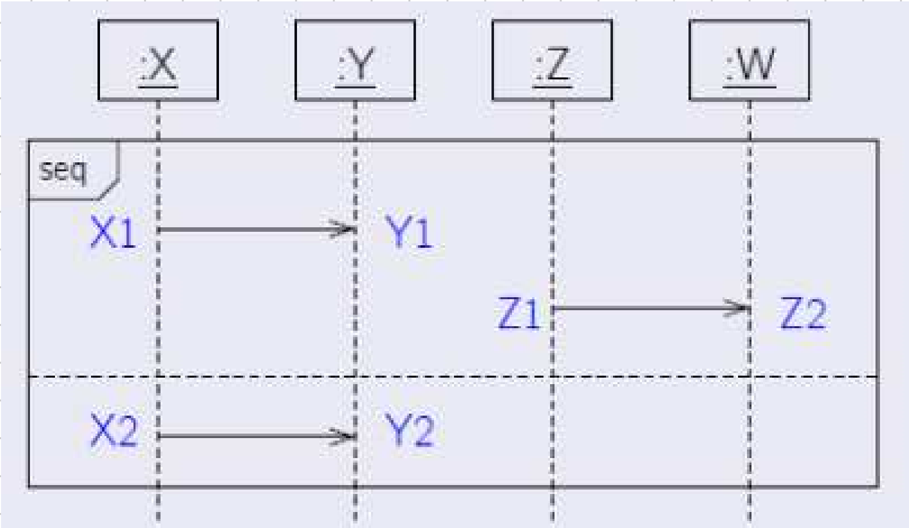
$X3, Y3, X1, Y1, Y2, X2$

$X1, X3, Y1, Y3, Y2, X2$

...




$$R = \{\underbrace{(X1,Y1),(Y1,Y2),(Y2,X2)}_{\leq_1}, \underbrace{(X3,Y3)}_{\leq_2}\}$$



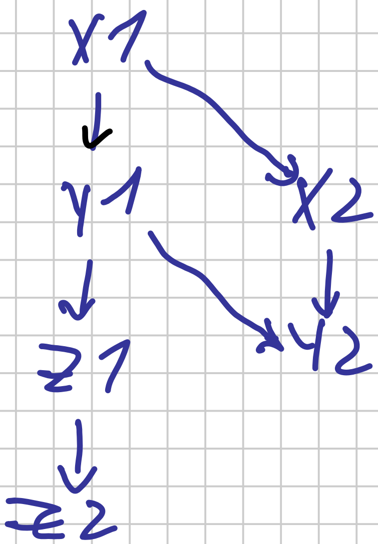

$X1 < Y1 < Z1 < Z2$

$X2 < Y2$

aufgrund von seq:

$X1 < X2$

$Y1 < Y2$




mögliche Abläufe!

X1, Y1, Z1, Z2, X2, Y2

X1, X2, Y1, Y2, Z1, Z2

X1, Y1, X2, Y2, Z1, Z2

X1, Y1, X2, Z1, Z2, Y2