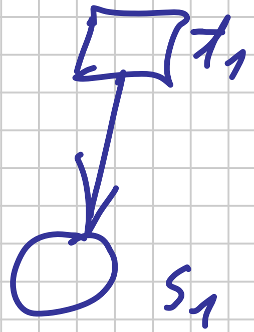
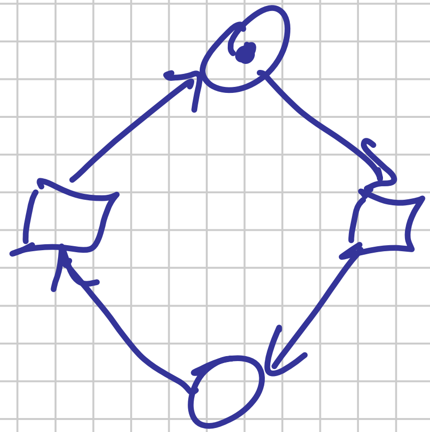
Modellierung 27.10.10

Notiztitel 27.10.2010

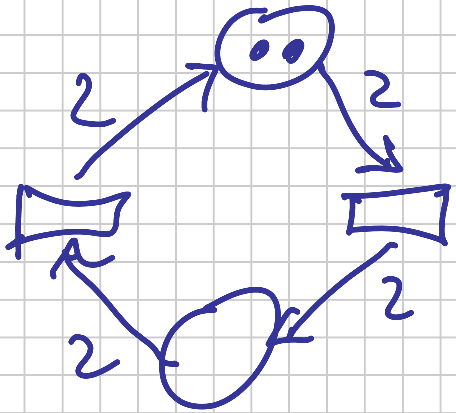


unbeschränkt

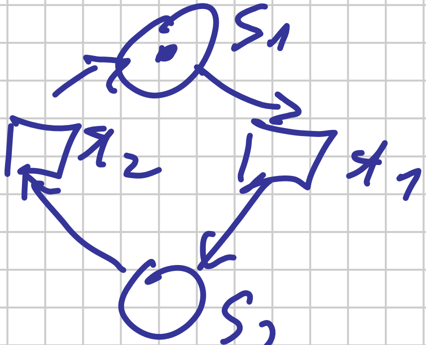
$$(0) \xrightarrow{t_1} (1) \xrightarrow{t_1} (2) \xrightarrow{t_1} (3) \xrightarrow{t_1} (4) \rightarrow \dots$$



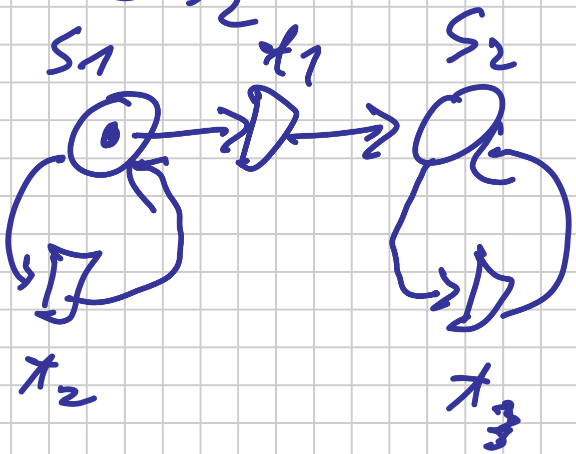
beschränkt, sicher



beschränkt, nicht sicher

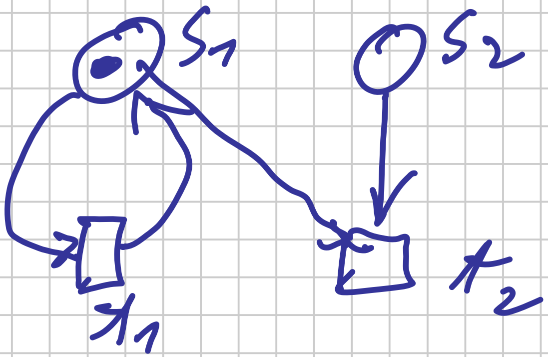


$$\begin{tikzcd} (1,0) \arrow[r, bend left, "t_1"] & (0,1) \arrow[l, bend left, "t_2"] \end{tikzcd}$$



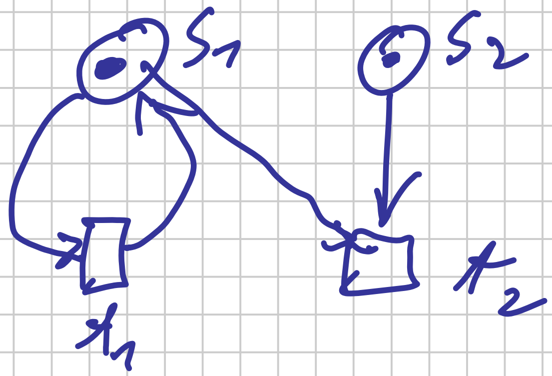
$$\begin{tikzcd} (1,0) \arrow[loop above, "t_2"] \arrow[r, "t_1"] & (0,1) \arrow[loop above, "t_3"] \end{tikzcd}$$

¬ stark lebendig
schwach lebendig
verklemmungsfrei



$$\begin{tikzcd} (1,0) \arrow[loop below, "t_1"] \end{tikzcd}$$

¬ schwach lebendig
verklemmungsfrei

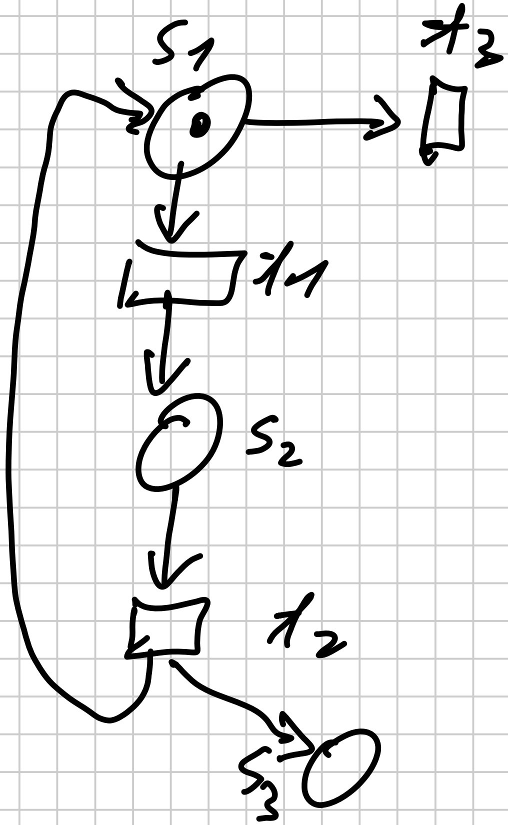


$$\begin{tikzcd}(1,1) \arrow[loop below, "t_1"] \arrow[r, "t_2"] & (0,0)\end{tikzcd}$$

$\neg$ stark lebendig

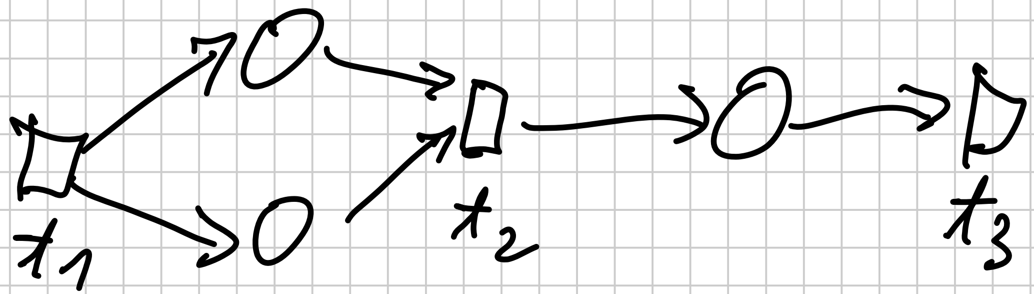
schwach lebendig

$\neg$ verklemmungsfrei



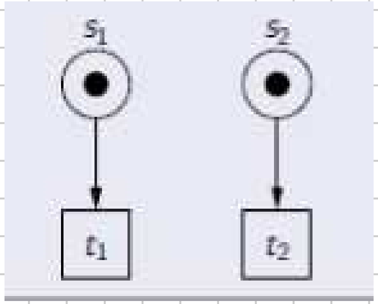
$$\begin{tikzcd}
{} \arrow[d] & \\
(1,0,0) \arrow[r, "t_3"] \arrow[d, "t_1"] & (0,0,0) \\
(0,1,0) \arrow[d, "t_2"] & \\
(1,0,\cancel{1}\,\omega) \arrow[r, "t_3"] \arrow[d, bend right, "t_1"'] & (0,0,\omega) \\
(0,1,\omega) \arrow[u, bend right, "t_2"'] &
\end{tikzcd}$$

Transitivität

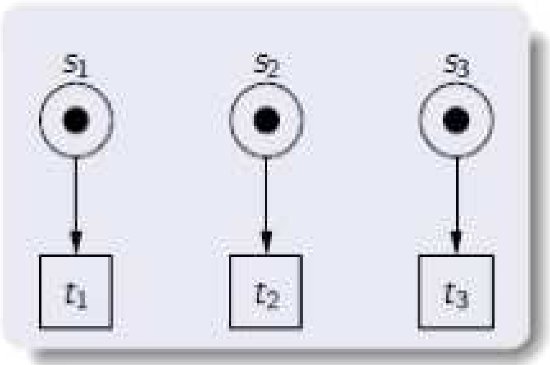


Modellierung 3.11.10

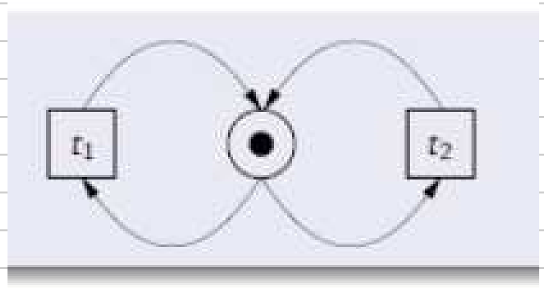
Notiztitel 03.11.2010



$$
\begin{tikzcd}
(1,1) \arrow[r, "t_1"] \arrow[d, "t_2"] & (0,1) \arrow[d, "t_2"] \\
(1,0) \arrow[r, "t_1"] & (0,0)
\end{tikzcd}
$$



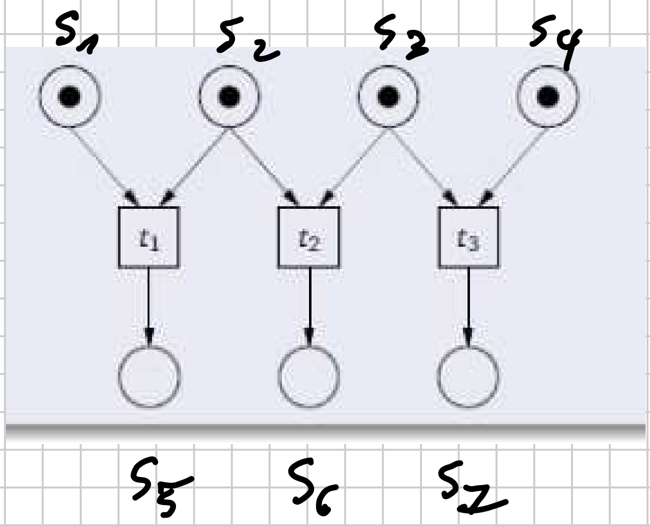
$$
\begin{tikzcd}
(1,1,1) \arrow[rr, "t_1"] \arrow[dr, "t_3"] \arrow[dd, "t_2"] & & (0,1,1) \arrow[dr, "t_3"] \arrow[dd, "t_2"] & \\
& (1,1,0) \arrow[rr, "t_1"] \arrow[dd, "t_2"] & & (0,1,0) \arrow[dd, "t_2"] \\
(1,0,1) \arrow[rr, "t_1"] \arrow[dr, "t_3"] & & (0,0,1) \arrow[dr, "t_3"] & \\
& (1,0,0) \arrow[rr, "t_1"] & & (0,0,0)
\end{tikzcd}
$$



$\bullet t_1 = (1)$

$\bullet t_2 = (1)$

$\bullet t_1 \oplus \bullet t_2 = (2) \not\leq (1)$

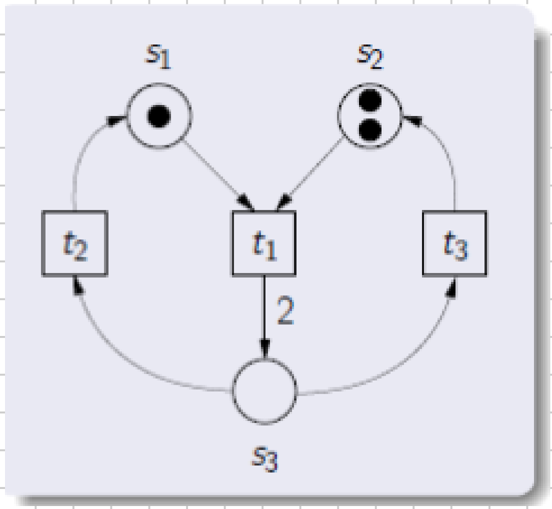


$$\vec{m} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad \vec{u} + \vec{v} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}$$

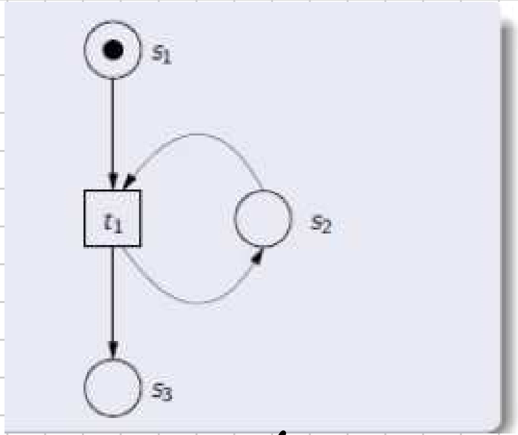
$$C \cdot \vec{u} = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} C_{11} \cdot u_1 + C_{12} \cdot u_2 + C_{13} \cdot u_3 \\ C_{21} \cdot u_1 + C_{22} \cdot u_2 + C_{23} \cdot u_3 \end{pmatrix}$$

$$u \cdot C = (u_1\ u_2) \cdot \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{pmatrix} = (u_1 C_{11} + u_2 C_{21},\ u_1 C_{12} + u_2 C_{22},\ u_1 C_{13} + u_1 C_{23})$$

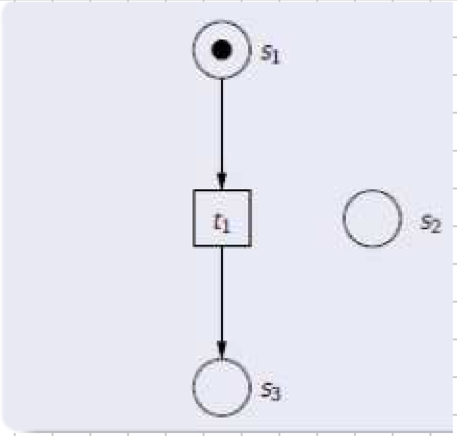


	t_1	t_2	t_3
s_1	0 - 1	1 - 0	0 - 0
s_2	0 - 1	0 - 0	1 - 0
s_3	2 - 0	0 - 1	0 - 1

$$C = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix}$$



$$\begin{array}{cc} & t_1 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} & \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \end{array}$$



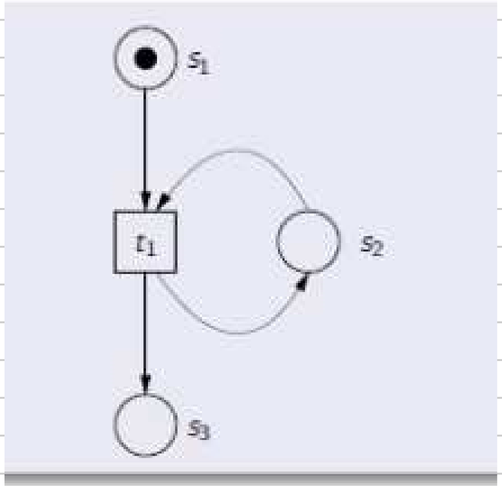
$$\begin{array}{cc} & t_1 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix} & \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

$$C = \begin{array}{cc} & \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ & \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} \end{array} \qquad \vec{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{array}{c} t_1 \\ t_2 \\ t_3 \end{array} \qquad \vec{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$C \cdot \vec{u}_1 = \begin{pmatrix} -1 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 \\ -1 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 \\ 2 \cdot 1 + (-1) \cdot 0 + (-1) \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \qquad C \cdot \vec{u}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

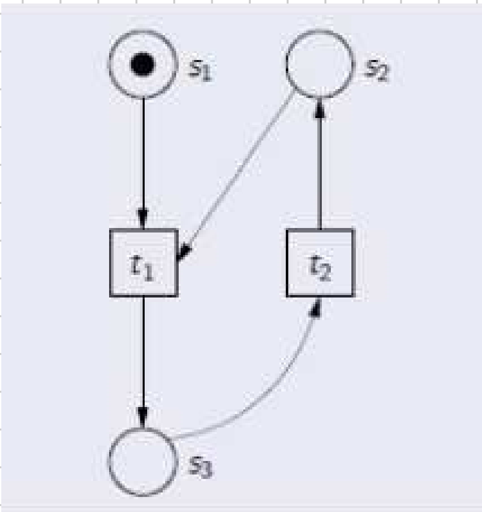
$$\vec{u} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \begin{array}{c} t_1 \\ t_2 \\ t_3 \end{array}$$

$$\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 2 + 1 \cdot 2 + 0 \cdot 1 \\ -1 \cdot 2 + 0 \cdot 2 + 1 \cdot 1 \\ 2 \cdot 2 + -1 \cdot 2 + -1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$



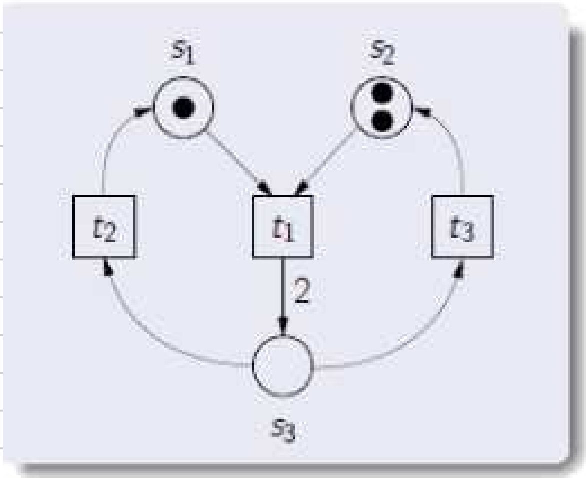
$$C = \begin{array}{c|c} & t_1 \\ s_1 & \\ s_2 & \\ s_3 & \end{array}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} (1) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$$C = \begin{pmatrix} -1 & 0 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + -1 \cdot 1 + 0 \\ 0 + -1 \cdot 1 + 1 \cdot 1 \\ 0 + 1 \cdot 1 + -1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$



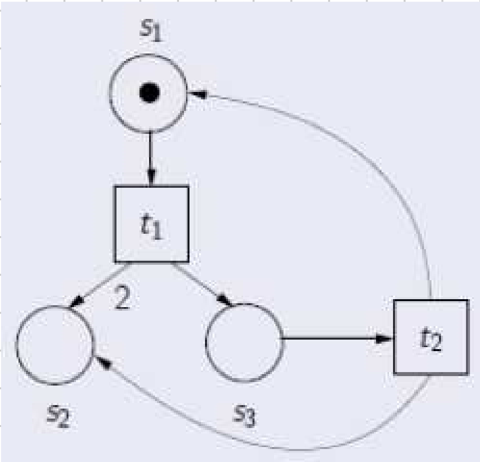
$$\begin{pmatrix}2\\2\\0\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \begin{pmatrix}-1 & 1 & 0\\-1 & 0 & 1\\2 & -1 & -1\end{pmatrix} \cdot \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$$

$$\begin{aligned}2 &= 1 - u_1 + u_2\\2 &= 2 - u_1 + u_3\\0 &= 0 + 2u_1 - u_2 - u_3\\\hline 4 &= 3 \quad \lightning\end{aligned}$$

Markierung $\begin{pmatrix}2\\2\\0\end{pmatrix}$ nicht erreichbar!

Modellierung 10.11.10

Notiztitel 10.11.2010



$$\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{array}{c c} & \begin{array}{cc} t_1 & t_2 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} & \begin{pmatrix} -1 & 1 \\ 2 & 1 \\ 1 & -1 \end{pmatrix} \end{array} \cdot \begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}1\\20\\0\end{pmatrix}$$

$$1 - u_1 + u_2 = 1$$

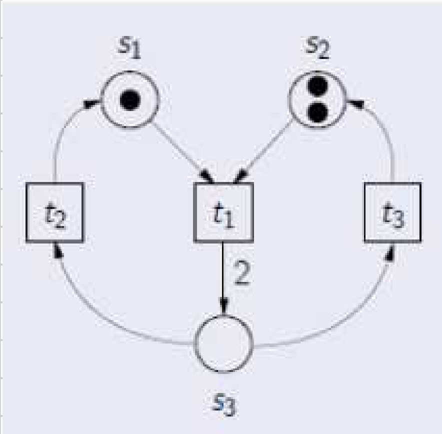
$$2u_1 + u_2 = 20 \quad | \quad 3u_1 = 20$$

$$u_1 - u_2 = 0 \quad | \quad u_1 = u_2$$

$$(1,0,0) \xrightarrow{t_1} (0,2,1) \xrightarrow{t_2} (1,\cancel{3}\,\omega,0)$$

$$(1,\omega,0) \xrightarrow{t_1} (0,\omega,1) \xrightarrow{t_2} (1,\omega,0)$$

T-Invariante



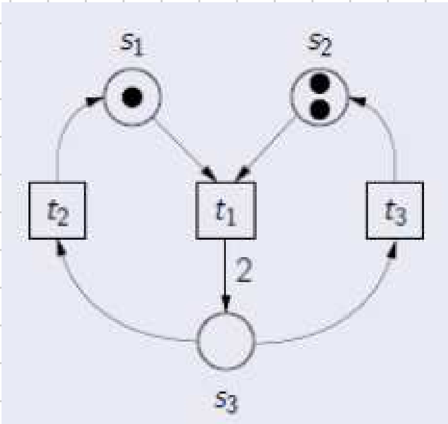
$$
\begin{array}{c|ccc}
 & t_1 & t_2 & t_3 \\
s_1 & & & \\
s_2 & & & \\
s_3 & & &
\end{array}
\quad
\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
$$

$$
\begin{aligned}
-u_1 + u_2 &= 0 \quad | \; u_1 = u_2 \\
-u_1 + u_3 &= 0 \quad | \; u_1 = u_3 = u_2 \\
2u_1 - u_2 - u_3 &= 0
\end{aligned}
$$

$$\vec{u} = k \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} k \\ k \\ k \end{pmatrix}$$

$$
\begin{tikzcd}
(1,2,0) \arrow[d, "t_1"] & \\
(0,1,2) \arrow[r, "t_3"] \arrow[d, "t_2"] & (0,2,1) \arrow[ul, bend right, "t_2"'] \\
(1,1,1) \arrow[uu, bend left, "t_3"] &
\end{tikzcd}
$$

S-Invariante



$$(\vartheta_1\ \vartheta_2\ \vartheta_3) \cdot \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} = (0\ 0\ 0)$$

$$-\vartheta_1 - \vartheta_2 + 2\vartheta_3 = 0$$

$$\vartheta_1 - \vartheta_3 = 0 \mid \vartheta_1 = \vartheta_3$$

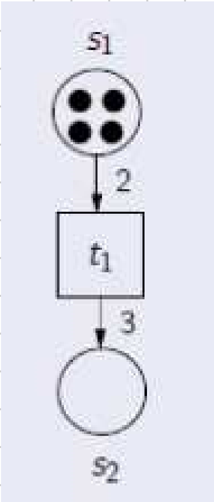
$$\vartheta_2 - \vartheta_3 = 0 \mid \vartheta_2 = \vartheta_3$$

$$\vartheta = (\vartheta_1\ \vartheta_1\ \vartheta_1) = \vartheta_1 \cdot (1\ 1\ 1)$$

$$\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

$$\vartheta = (1\ 1\ 1)$$

$$\vartheta \cdot \vec{m} = m_1 + m_2 + m_3 = 3 = \vartheta \cdot m_0$$



$$(v_1, v_2) \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} = (0)$$

$$-2v_1 + 3v_2 = 0$$

$$v_1 = \frac{3}{2} v_2$$

$$v = v_2 \cdot \left(\frac{3}{2} \quad 1\right)$$

$$v = \ell \cdot (3 \quad 2)$$

$$v \cdot \vec{m} = 3 \cdot m_1 + 2 \cdot m_2 = 12 = v \cdot \vec{m_0}$$

$$m = (0, 5)$$

$$v \cdot m = 3 \cdot 0 + 2 \cdot 5 = 10 \neq 12$$

Modellierung 17.11.10

Notiztitel 17.11.2010

$$v \cdot \begin{array}{c|cccc} & t_1 & t_2 & t_3 & t_4 \\ \hline s_1 & 1 & -1 & 0 & 0 \\ s_2 & 0 & -1 & 1 & 0 \\ s_3 & -1 & 1 & 0 & 0 \\ s_4 & 0 & 1 & -1 & -1 \\ s_5 & 0 & 0 & 0 & 1 \end{array}$$

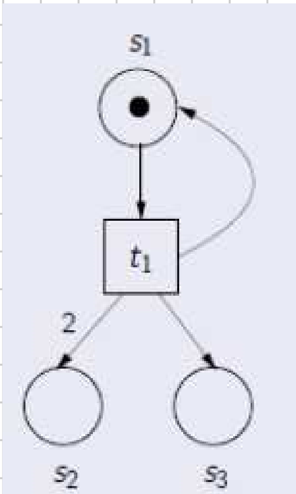
$$\begin{aligned} v_1 - v_3 &= 0 \quad | \; v_1 = v_3 \\ -v_1 - v_2 + v_3 + v_4 &= 0 \\ v_2 - v_4 &= 0 \quad | \; v_4 = v_2 \\ -v_4 + v_5 &= 0 \quad | \; v_4 = v_5 \end{aligned}$$

$v_1 = v_3$, $v_2 = v_4 = v_5$

$$v = (a\, b\, a\, b\, b) = a(1\,0\,1\,0\,0) + b(0\,1\,0\,1\,1)$$

$$v \cdot \vec{m} = (a\, b\, a\, b\, b) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = a + 2b \quad \Big| \quad \vec{v} \cdot \vec{m}_0 = (a\, b\, a\, b\, b) \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} =$$

$\neq$ $\rightarrow a + b$



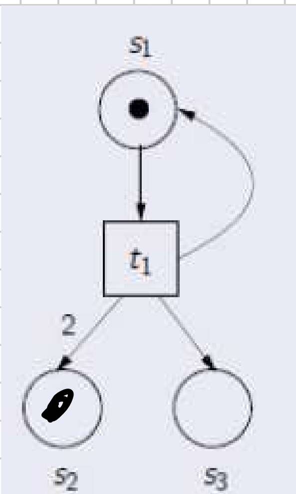
$$(v_1, v_2, v_3) \cdot \begin{array}{c|c} & t_1 \\ \hline s_1 & 0 \\ s_2 & 2 \\ s_3 & 1 \end{array} = 0 \Rightarrow 2v_2 + v_3 = 0$$

$$v_3 = -2v_2$$

$$v = \ell(0 \;\; 1 \; -2)$$

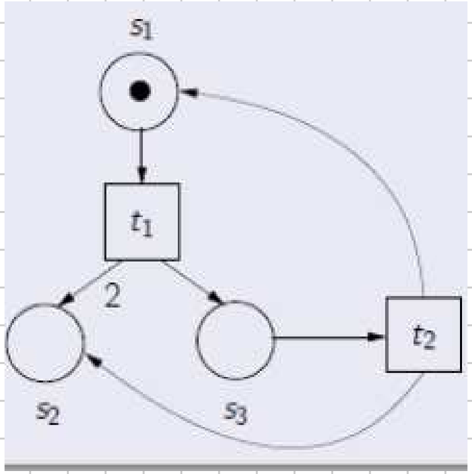
$$v \cdot \vec{m} = 0m_1 + m_2 - 2m_3 = 0 \Rightarrow m_2 = 2m_3$$

$$v \cdot \vec{m}_0 = 0$$



$$v \cdot \vec{m}_0 = 0m_1 + m_2 - 2m_3 = 1 = v \cdot \vec{m}$$

für alle erreichb. Mark. $m_2 = 2m_3 + 1$



$$\begin{array}{c|cc} & t_1 & t_2 \\ \hline s_1 & -1 & 1 \\ s_2 & 2 & 1 \\ s_3 & 1 & -1 \end{array}$$

$$-v_1 + 2v_2 + v_3 = 0$$
$$v_1 + v_2 - v_3 = 0$$
$$\overline{\qquad v_2 = 0 \qquad}$$
$$v_1 = v_3$$

$$v = l \cdot (1\,0\,1)$$

$$v \cdot \vec{m}_0 = (1\,0\,1)\begin{pmatrix}1\\0\\0\end{pmatrix} = 1$$
$$\|$$
$$v \cdot \vec{m} = (1\,0\,1)\begin{pmatrix}1\\2\;0\\0\end{pmatrix} = 1$$

gemäß S-Invariante erreichbar, tatsächlich aber nicht

$$\begin{array}{c|cccc} & t_1 & nt_1 & t_2 & nt_2 \\ \hline k_1 & & & & \\ Nk_1 & & & & \\ k_2 & & & & \\ Nk_2 & & & & \\ S & & & & \end{array} \quad \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

S-Invarianten

$$v_1 - v_2 \qquad - v_5 = 0$$
$$-v_1 + v_2 \qquad + v_5 = 0$$
$$v_3 - v_4 - v_5 = 0$$
$$-v_3 + v_4 + v_5 = 0$$

$$\left.\begin{aligned} v_1 &= v_2 + v_5 \\ v_3 &= v_4 + v_5 \end{aligned}\right\} \quad (v_2+v_5,\ v_2,\ v_4+v_5,\ v_4,\ v_5)$$

$$= v_2 \cdot (1,1,0,0,0) + v_4 \cdot (0,0,1,1,0) + v_5 \cdot (1,0,1,0,1)$$

wähle $v_2 = v_4 := 0$ $\quad (1\,0\,1\,0\,1) \cdot \vec{m} = m_1 + m_3 + m_5 = 1 = v \cdot \vec{m}_0$

Annahme: $m_1 \geq 1,\ m_3 \geq 1 \Rightarrow v \cdot \vec{m} = m_1 + m_3 + m_5 \geq 2$

↯ Widerspruch

Speisende Philosophen

eine der S-Invarianten

$$v = \begin{array}{cccccccc} (1 & 0 & 0 & 0 & 1 & 1 & 1 & 1) \\ F_1 & F_2 & H_1 & H_2 & W_1 & W_2 & E_1 & E_2 \end{array}$$

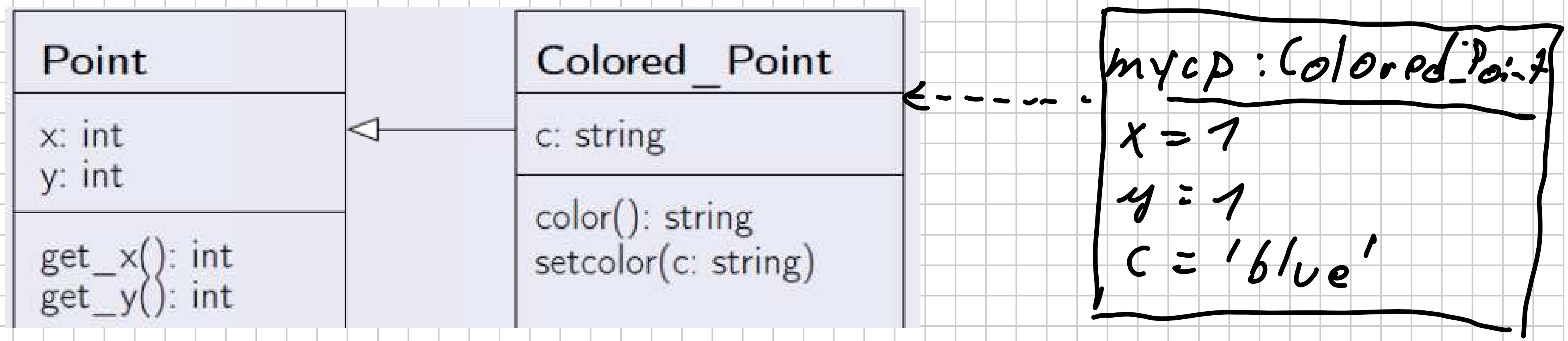
$v \cdot \vec{m}_0 = 1$

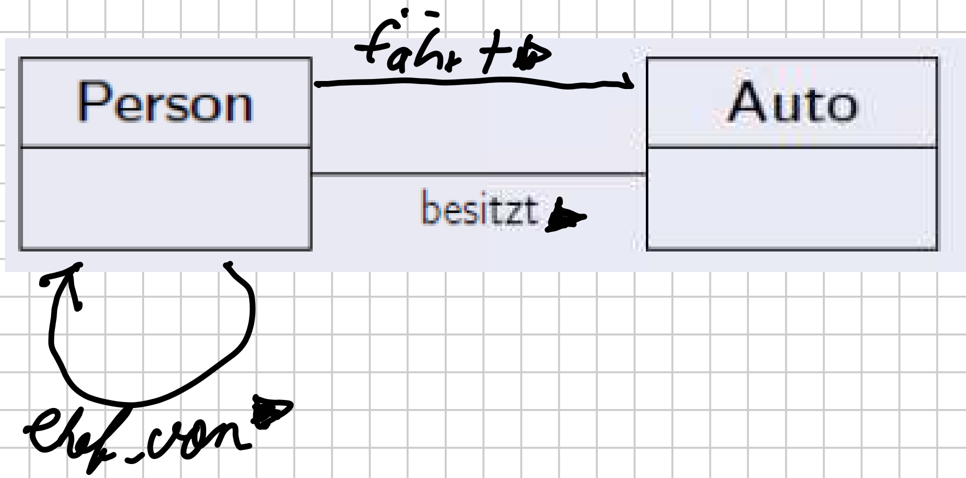
Verklemmung: Beide in W_1 bzw. W_2 $v \cdot \vec{m} = 2 \neq 1$

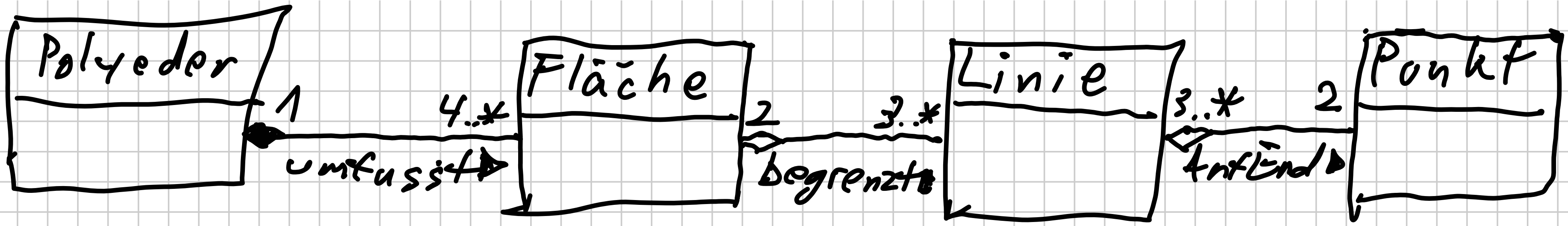
$\Rightarrow$ Verklemmung kann nicht auftreten

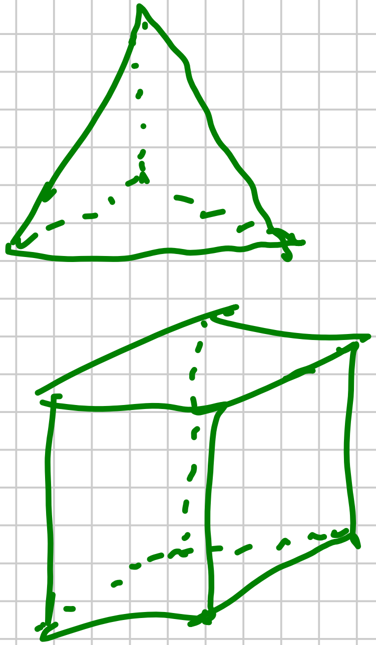
Modellierung 1.12.10

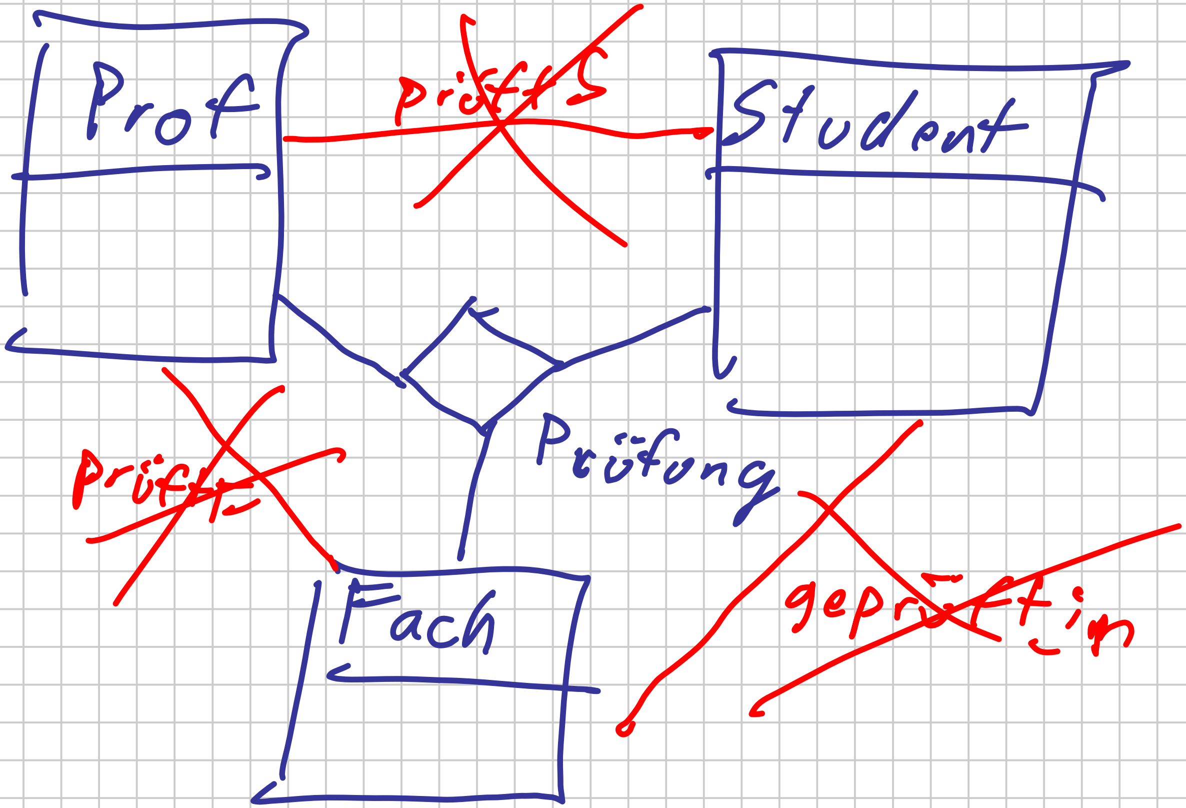
Notiztitel 01.12.2010

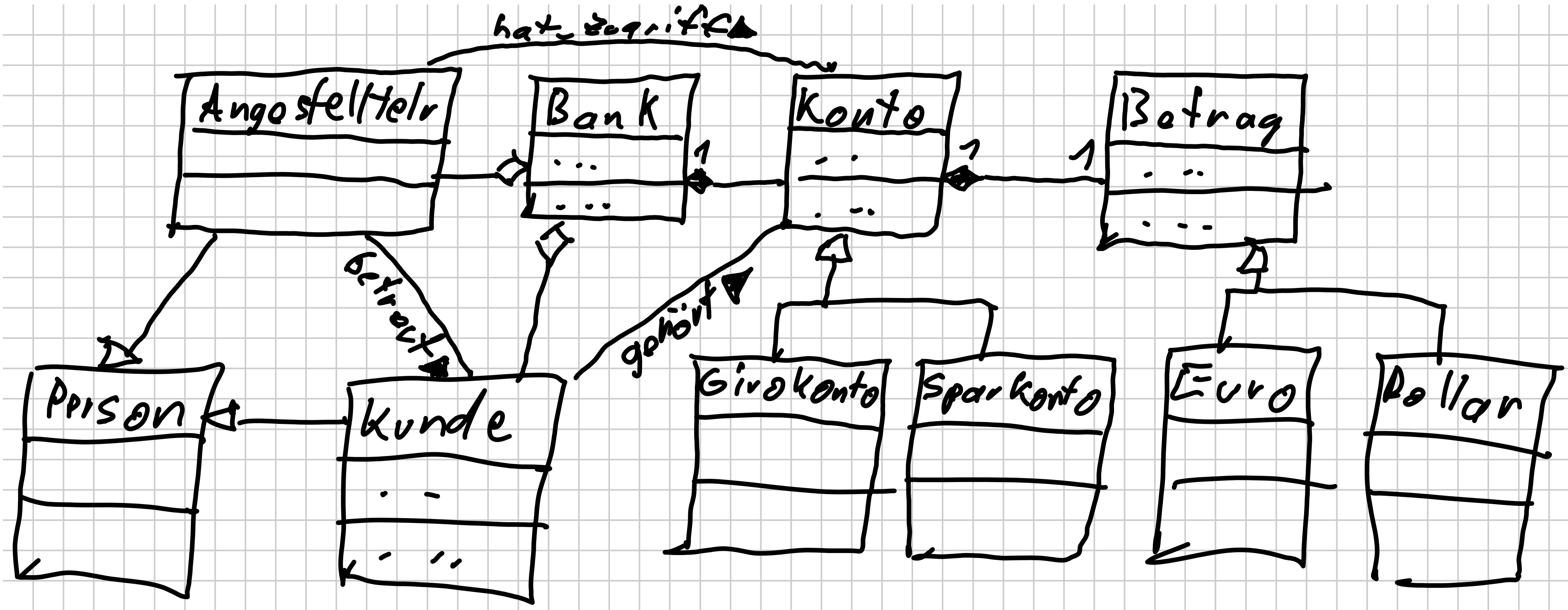





Polyeder
1
umfasst
4..*
Fläche
2
begrenzt
3..*
Linie
3..*
2
Punkt



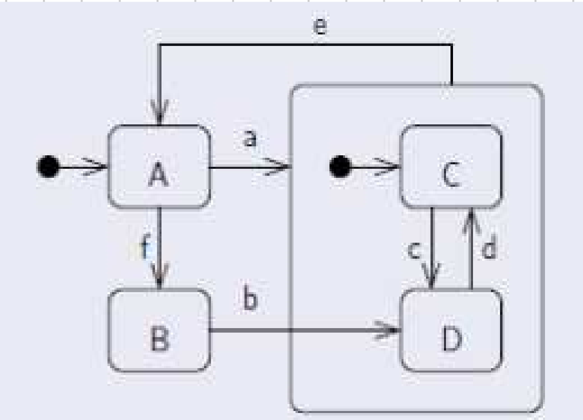

Prof
~~prüft~~
Student
~~prüft~~
Prüfung
Fach
~~geprüft in~~

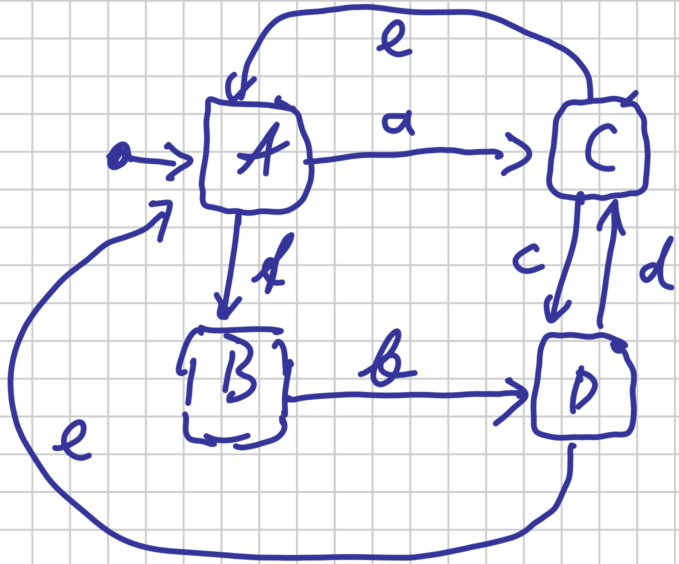


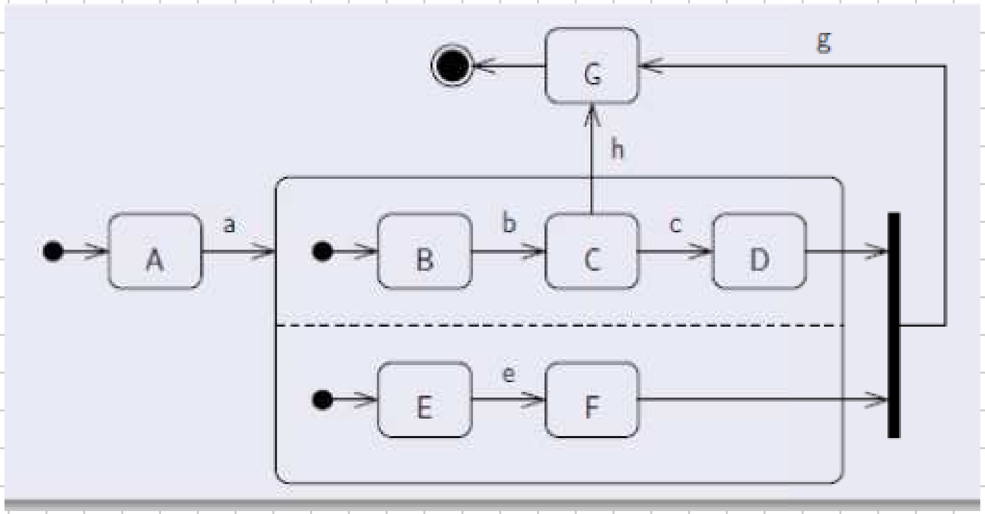
hat_zugriff
Angestellter
Bank
1
Konto
1
1
Betrag
betreut
gehört
Person
Kunde
Girokonto
Sparkonto
Euro
Dollar

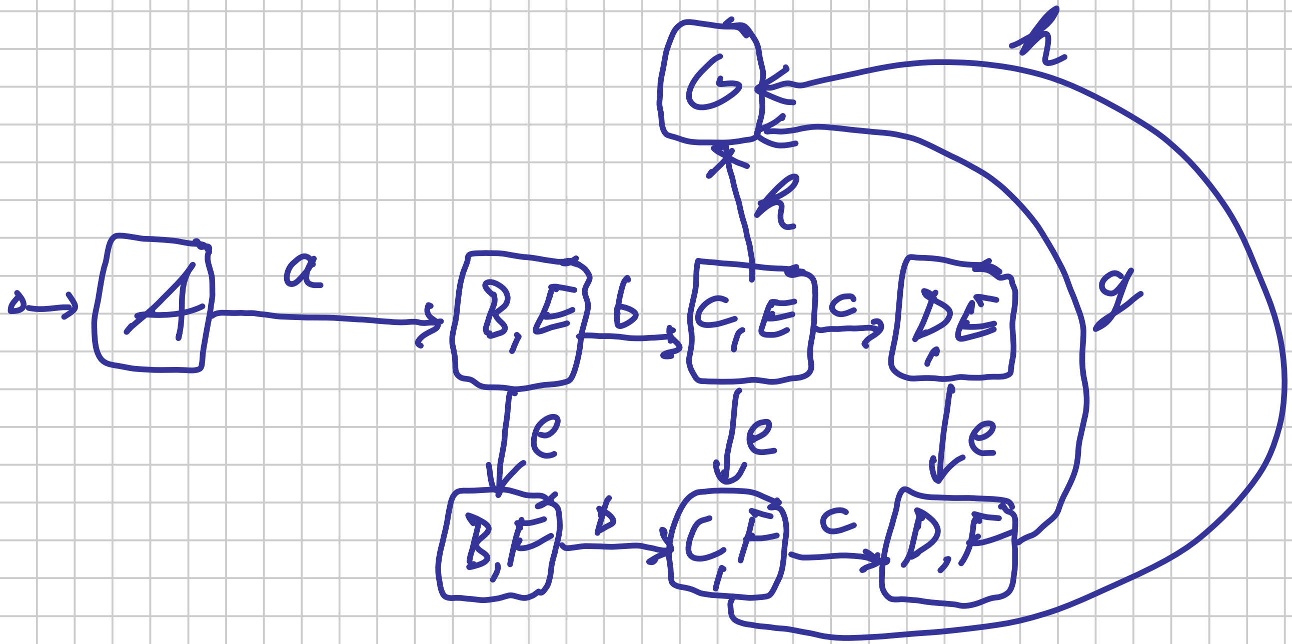
Modellierung 22.12.10

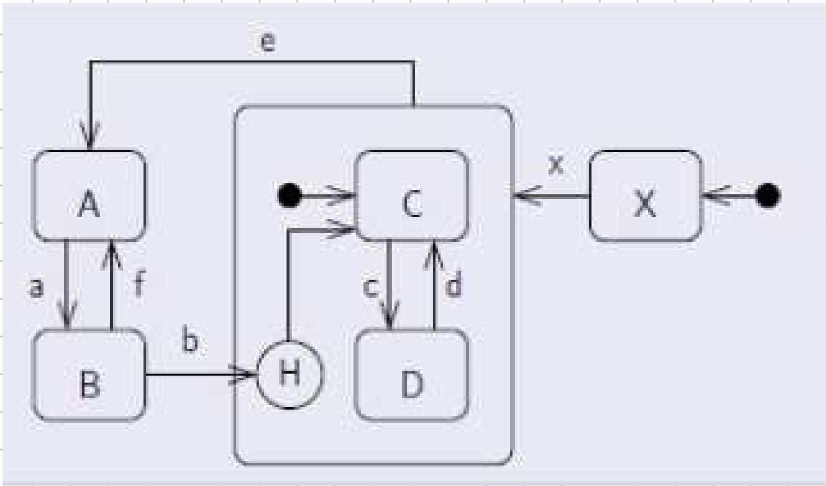
Notiztitel 22.12.2010

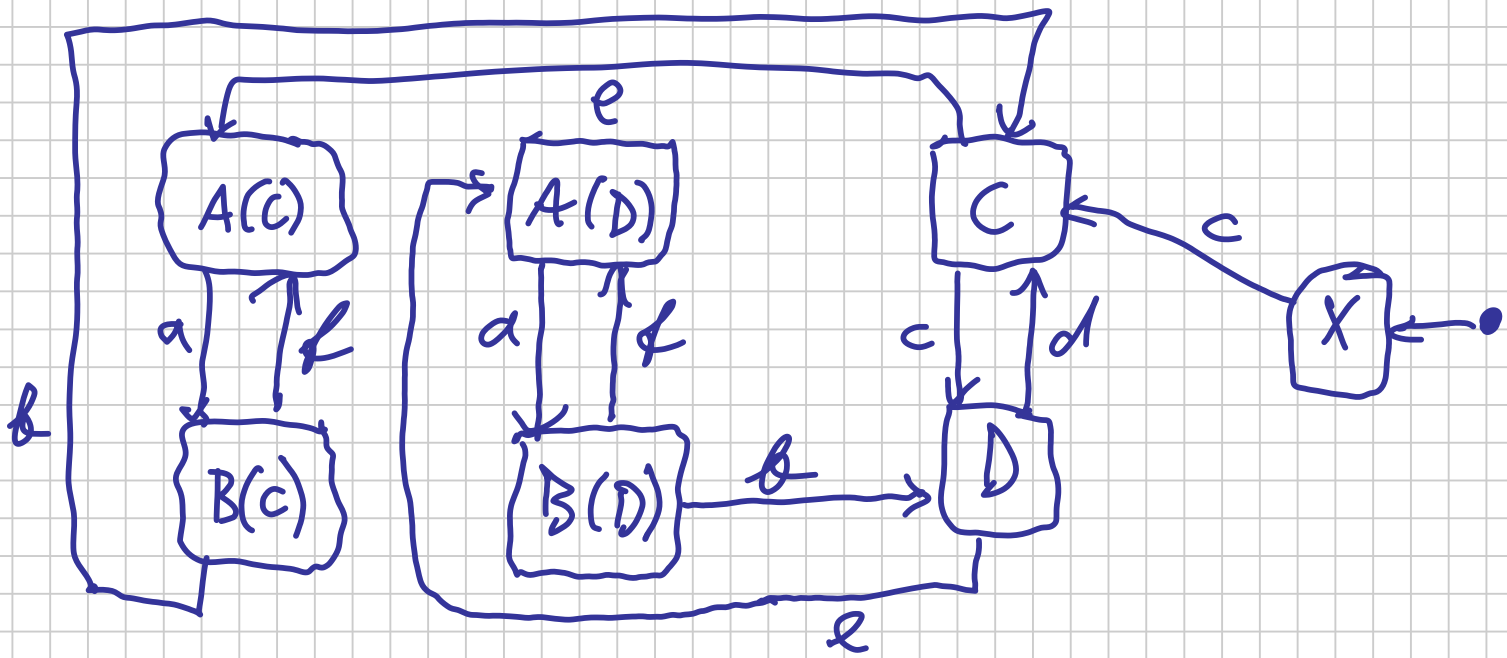





A
a
B
b
C
c
D
h
G
g
E
e
F

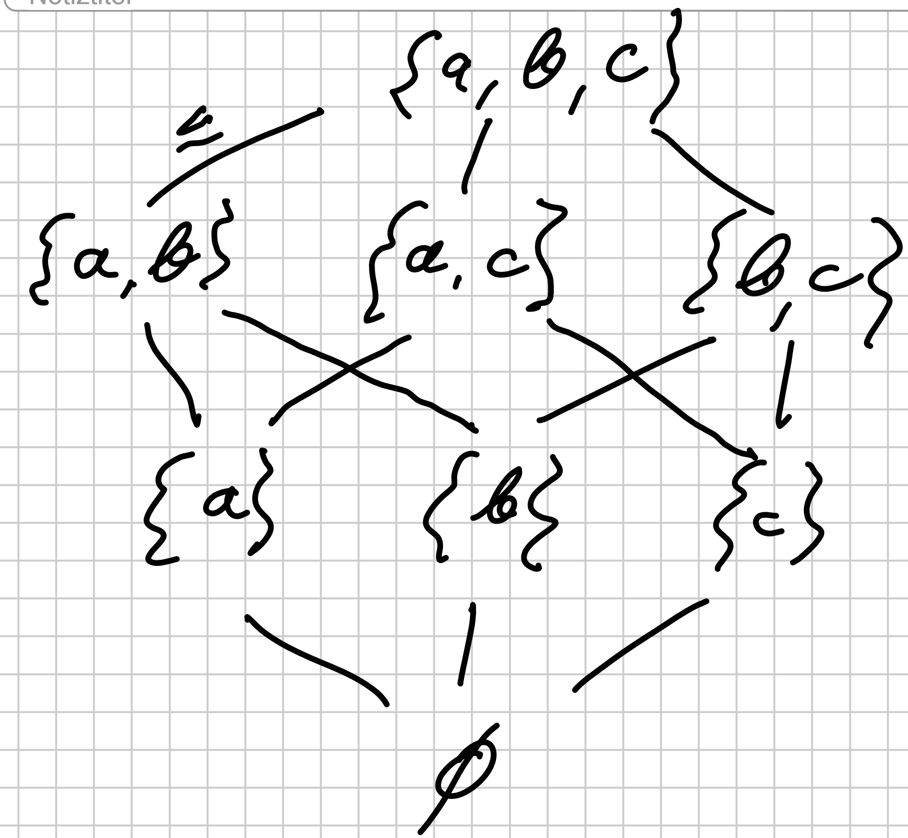

A
a
B,E
b
C,E
c
D,E
e
B,F
C,F
D,F
h
G
g


e
A
a
f
B
b
H
C
c
d
D
x
X


A(C)
B(C)
a
f
b
A(D)
B(D)
e
C
D
c
d
X

Modellierung 12.1.11

Notiztitel 12.01.2011



Partielle Ordnung:

$\subseteq$ auf Mengen

Reflexivität: $M \subseteq M$

Transitivität:

$A \subseteq B, B \subseteq C \Rightarrow A \subseteq C$

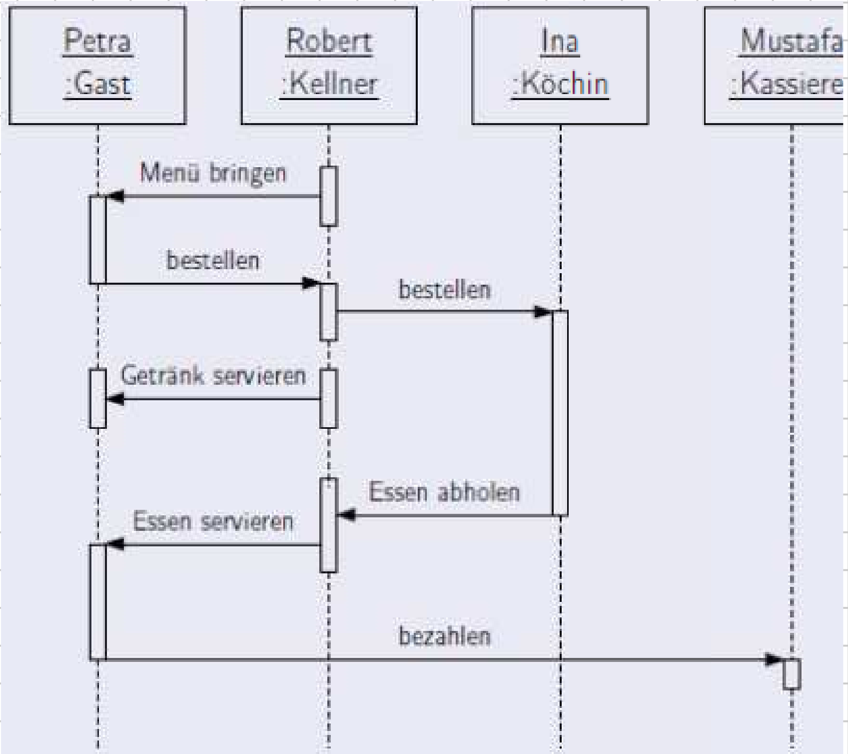
Antisymmetrie:

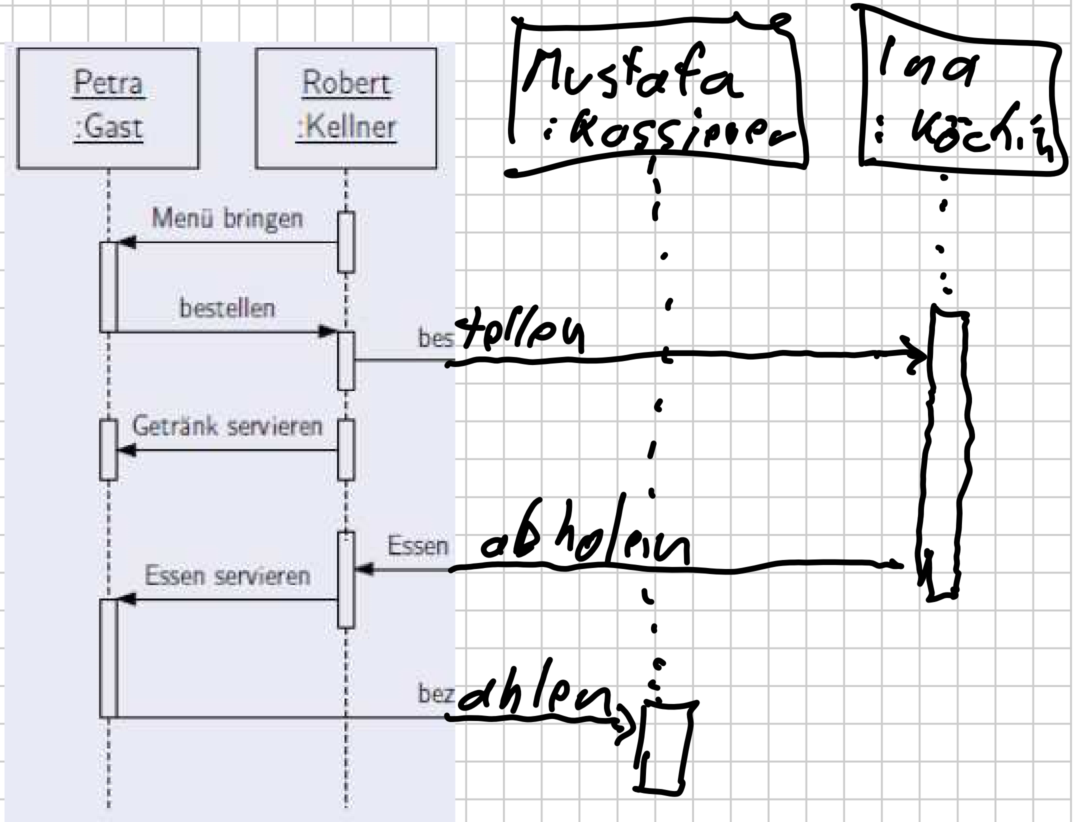
$A \subseteq B, B \subseteq A \Rightarrow A = B$

$R = \{(1,2),(2,3),(3,4),(3,5)\}$

$$\begin{aligned} R^* = \{&(1,2),(2,3),(3,4),(3,5),\\ &(1,3),(1,4),(1,5),\\ &(2,4),(2,5),(1,1),(2,2),\\ &(3,3),(4,4),(5,5)\} \end{aligned}$$

$$\begin{tikzcd}
 & & 1 \arrow[dl] \\
 & 2 \arrow[d] & \\
 & 3 \arrow[dl] \arrow[dr] & \\
4 & & 5
\end{tikzcd}$$


Petra
:Gast
Robert
:Kellner
Ina
:Köchin
Mustafa
:Kassiere
Menü bringen
bestellen
bestellen
Getränk servieren
Essen abholen
Essen servieren
bezahlen


Petra
:Gast
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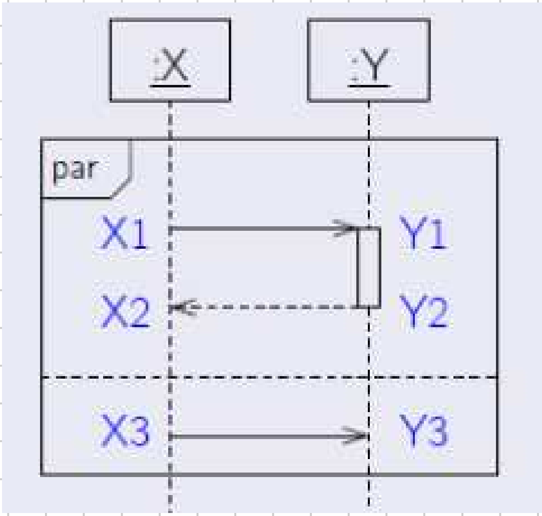


Ordnung auf den Ereignissen

$X1 < Y1 < Y2 < X2$

$X3 < Y3$

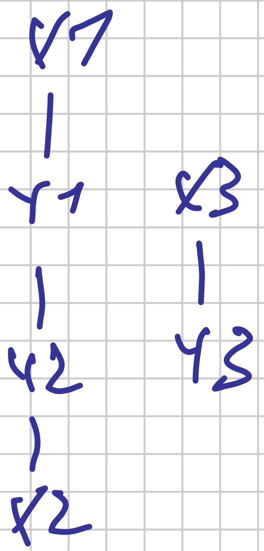
mögliche Abläufe:

$X1, Y1, Y2, X2, X3, Y3$

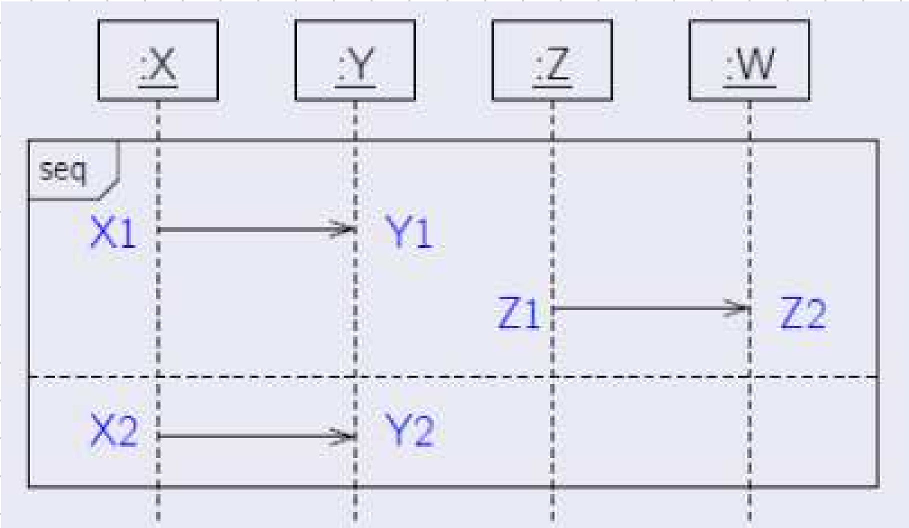
$X3, Y3, X1, Y1, Y2, X2$

$X1, X3, Y1, Y3, Y2, X2$

...



$$R = \{\underbrace{(X1,Y1),(Y1,Y2),(Y2,X2)}_{\leq_1},\underbrace{(X3,Y3)}_{\leq_2}\}$$



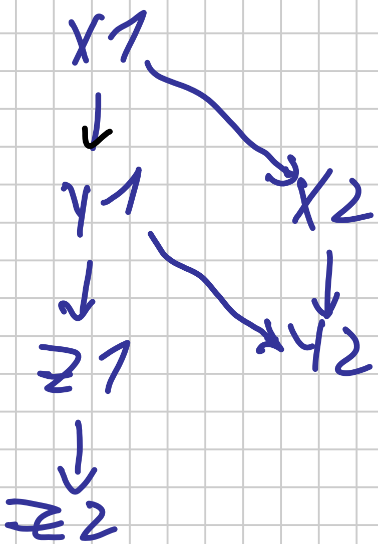
$X1 < Y1 < Z1 < Z2$

$X2 < Y2$

aufgrund von seq:

$X1 < X2$

$Y1 < Y2$



mögliche Abläufe!

X1, Y1, Z1, Z2, X2, Y2

X1, X2, Y1, Y2, Z1, Z2

X1, Y1, X2, Y2, Z1, Z2

X1, Y1, X2, Z1, Z2, Y2